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Luca D'Acci
Editor

The Mathematics of Urban Morphology

Foreword by Michael Batty

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Editor
Luca D'Acci
Interuniversity Department of Regional
and Urban Studies and Planning
Politecnico di Torino
Torino, Italy

Erasmus School of Social and Behavioural
Sciences
Erasmus Universiteit Rotterdam
Rotterdam, The Netherlands

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Foreword: The Morphology of Cities

I once took the ICE train from Frankfurt to Weimar to give a talk about cellular automata modelling to architectural students at the original Bauhaus. In Weimar, I not only visited the buildings where Walter Gropius set up his school that fostered the modern movement, encapsulated in Louis Sullivan's hallowed phrase 'form follows function', but I also discovered that it was in Weimar where Goethe spent most of his life, writing of many things but in particular about 'form', exploring the way plants grew and metamorphized. In 1817, there he wrote: "Form is a thing in motion, in the process of becoming, of passing away. The study of form is the study of transformation. The study of metamorphosis is the key to all the signs of nature." (J. W. von Goethe, *Zur Morphologie*, Stuttgart, Germany: J. G. Cotta Publishers, 1817, 201).

Form follows function has been the dominant mantra not only for modern architecture throughout most of the twentieth century, but for urbanism too and most historical studies of city form are structured according to the notion that cities develop around markets where there is competition to trade goods and where nearness to these points of exchange is more valued than locations further away. Thus, cities are denser nearer their centres and their form thus represents a balance between centralizing forces that concentrate on activities and decentralizing forces that attempt to capture more space, thus diffusing activities as far from their centres as possible. This, then, is the economic trade-off that determines the form of the city with respect to the juxtaposition of all its key activities.

Traditionally, the form of the city is one whose descriptions are both verbal and visual. In his introductory essay which follows next, the editor Luca D'Acci takes several quotes from Italo Calvino's *Invisible Cities* (Harcourt, Brace, and Jovanovich, 1978), which is a speculation on ideal cities but expressed entirely verbally. In contrast, many works focus on the visual structure of cities, often as maps and plans, but also linking to the third dimension and betraying a strong architectural and aesthetic interest that has been the dominant force in the descriptive study of urban morphology (for example see Spiro Kostoff's books *The City Shaped*, Thames and Hudson, 1993 and *The City Assembled*, Thames and Hudson, 1999). Exploring urban morphology using more abstract and mathematical

methods, however, apart from occasional forays into the geometry of cities in the nineteenth century, did not begin until the late 1950s when M.R.G. Conzen first began to study the geographic structure of small towns at the level of plots and streets. In the 1970s, various approaches coming from the formal study of building layouts, as for example, in work from the Centre for Land Use and Built Form Studies (see Lionel March and Philip Steadman's wonderful statement of this new movement in *The Geometry of Environment*, RIBA Press, 1971), provided another twist to these perspectives. Thinking about the geometric structure of cities as it was reflected in the design process also generated another theme in the serious development of urban morphology. Christopher Alexander's contributions, particularly his book *Notes on the Synthesis of Form* (Harvard, 1964) strongly define this tradition.

The mathematical approach, however, did not really make itself felt until about 30 years ago and this followed in the wake of complexity theory. In the 1950s and 1960s, the systems approach became fashionable and although this was entirely consistent with formal and computational approaches to cities and their design, the city itself resisted any formal theories other than those that were developed within location theory and urban economics. The idea of form was thus implicit in these largely aggregate spatial theories. The problem was partly that the systems approach focused on thinking of cities as machines, to be fashioned and planned from the top-down, whereas more generally in science from that time, the idea that systems evolved from the bottom up was beginning to make itself felt. Essentially cities (and many other kinds of social system—the economy, for example) came to be thought of as more like organisms than machines. Biology thus began to replace physics as the dominant metaphor and the notion that cities largely evolve rather than being planned in any sense, took hold. In other words, complexity theory emerged and it was this that introduced the new thinking about urban morphology of which this book is a statement.

Much of what has developed since then which can be badged rather loosely under the label of mathematical morphology, is illustrated in the collection of articles gathered together here. The book is largely structured around the main key approaches. The properties of cities with respect to scale and size are introduced in diverse ways and new geometries based on fractals constitute the first part of the book, where form and function using ideas from statistical physics are presented. Transformations of form in cities, of course, are key to complex systems and the processes of generating forms using ideas from cellular automata follow these first chapters on fractals. The focus then changes to networks. There are a number of chapters on elementary methods of looking at cities as network structures as in space syntax, where there is a slow transition to ideas that are emerging in network science. The emphasis here, however, is on streets as networks rather than on clusters and one of the slowly emerging themes in this whole domain involves how networks and clusters might be reconciled—in particular through ideas such as percolation theory.

The wider context of complexity theory as a frame of reference for studies of urban morphology is then presented and this is followed by a series of studies which do not fit into the classic frame of fractals, cellular automata or statistical physics. These are then followed by commentaries, vignettes almost, some of which recall historical developments, others of which are more speculative summaries of the field, defining further directions for research. In fact, although the urban morphological work of M.R.G. Conzen in the 1960s was the first of the formal approaches to be developed in this broad domain, it is in this last section that the Conzenian tradition is best explained, in fact by his son and others. In fact, the younger Conzen's own contribution to this book presented in the last part focuses on how we might integrate the many approaches developed here with his father's tradition and those of his colleagues such as Whitehand. However, the major part of the book deals with different approaches but the last part introduces speculations on the future of the field and on ways in which these ideas can be connected to developments in computing, transport and related features of the smart city that are energizing the field. The collection of essays here is quite rounded and although there are areas of morphology that are not covered in detail, all the pointers to the wider field mean that the diligent reader can easily navigate the broad domain to which the contributions here ascribe.

In the first part which is devoted to fractals, the key focus is on how self-similar structures that reflect the hierarchy of central places and of streets—that is of locations and interactions, nodes and networks—sort themselves out through competition. The results of any competitive process result in the rich getting richer and the poor getting poorer. If a lower bound is put on poverty which might be a basic form of existence or survival even, then what happens is that the competitive process which is essentially random, generates power laws which relate the frequency of occurrence of an object with its size. All the papers in this section end up showing how the classic signatures of fractal morphology are power laws, often stated in the form of the counter-cumulative frequency distribution of city (and other) sizes, which is usually called Zipf's Law. This applies across the board, from the way streets are ordered to the correlations between clusters of self-similar development. Chen leads the way with a classic demonstration of correlation and fractals. Hsu and Zou illustrate rather nicely how power laws arise quite naturally from ideas about central places, while Bee and his colleagues show how Zipf, Gibrat and Pareto can all be related as power laws. Neillson and Gil do the same for streets also measuring organic growth, while Mengyuan and colleagues illustrate that for Brisbane, urban form is less fractal, meaning less circuitous than other city examples from around the world. This leads rather easily to a short second part where cellular automata models are introduced by Antoni and his colleagues, these generating fractal forms, while the section is concluded by a foray into the origins of cellular automata with respect to self-replicating forms which lie at the basis of computation, presented by Clarke.

The third part deals with networks, essentially street networks although the ideas might be applied to other forms. Volchenkov introduces the mathematics of networks and then there is a long chapter which summarizes space syntax by Mahbub

that, in turn, is followed by a summary of applications of syntax by van Nes. Boeing then examines some very basic proprieties of networks, in particular the extent to which actual walking networks are more efficient than driving networks. He thus raises the thorny issue of how circuitous networks should be, thus implying that networks built out of straight-line segments are too simple to represent the properties of compactness and directedness in cities. In the fourth part, three contributions stand back a little from the technical expositions that dominate the book thus far, and these deal with complexity, per se. Goh and colleagues introduce emergence of complexity in urban morphology, focussing on generative models and these relate to those in the first part. Bellomo and Terna look at transformations and metamorphosis, following Goethe to an extent and invoking images from Escher. Jiang completes the section looking at Christopher Alexander's ideas of 'wholeness' with respect to entire city systems, where he focuses more on topological properties, but without losing ideas about hierarchy.

In the fifth part, a series of interesting but somewhat ad hoc set of ideas about mathematical morphology are introduced. Schirmer and Axhausen examine multiple scales directly and these have powerful links to earlier papers. In fact, most papers deal with development and clusters on one scale only and although many scales are dealt with, most of the contributions do not compare data or outcomes across scales. Raimbault then deals with interactions between networks and locations, while Huynh introduces percolation and point pattern analysis. Marshall and colleagues deal with compactness and Li and Ratti show how to extract morphologies from *Google Street View* in the form of street canyons and related volumes. Last but not least, spatial interaction models are introduced for the first time by Burger and colleagues and this serves to tie together key ideas involving locations, networks and clusters of development.

The book then moves to commentaries that provide a rather refreshing foil to the technical essays that dominate most of the book. Roger White first argues that in cellular modelling, which one might take as being morphological modelling, it is algorithms not mathematics that count. This is quite a controversial point of view but it reflects a number of key paradoxes and conflicts between different approaches which are also echoed in Trevor Barnes' ideas that we should begin to work hard on integrating mathematical models and urban social theory. Ron Johnston introduces his own experience with Conzenian urban morphology and its wider relation to social area analysis, and Peter Larkham takes a wide view of how one might think about integrating complexity ideas, fractals, cellular automata and networks into these traditional morphological approaches. Chris Rogers argues that infrastructure and engineering issues should be integrated with these ideas while Carola Hein and Tino Mager maintain that we need a deeper discussion of computation in the context of humanistic systems such as cities. Vitor Oliveira links agent-based models to traditional morphology and then Diane Davis and Andres Sevtsuk look at what might happen to streets when we move to an era of automated vehicles. Steffen Lehmann then argues that density is the key to the future city, thus echoing many of the contributions to this volume.

In this last part, Michael Conzen's commentary requires particular mention. Like Ron Johnston and Trevor Barnes, he augments his father's and his own approach which we have referred to here as a more descriptive geographical perspective on morphology, with a call for integrating the diversity of approaches presented here. His clear and balanced discussion of the way urban morphology has expanded its boundaries as a focus of study provides a useful synthesis of where the field is heading and thus provides a fitting point to the future. To reinforce his argument, he issues a challenge for the field: to develop the many alternative approaches on a single place, so a true comparative analysis might be developed. Many insights could flow from this. Such a project has been attempted before in comparing different land use transport models for the same place and there are many pitfalls. But considerable learning about how far each approach enriches our understanding of cities would be the result.

This book provides a useful perspective on the state of the art with respect to urban morphology in general and mathematics as tools and frames to disentangle the ideas that pervade arguments about form and function in particular. There is much to absorb in the pages that follow and there are many pointers to ways in which these ideas can be linked to related theories of cities, urban design and urban policy analysis as well as new movements such as the role of computation in cities and the idea of the smart city. Much food for thought. Read on, digest, enjoy.

London, UK

Michael Batty
The Bartlett School of Planning, CASA
University College London (UCL)

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