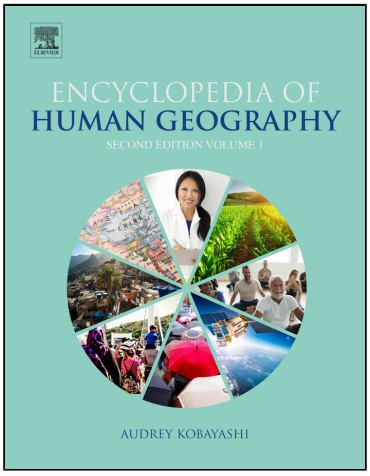
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Chaos and Complexity☆

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Glossary

Chaos The deterministic behavior of a dynamic system in which no system state is ever repeated. In terms of the phase space, this means that no point is ever revisited.

Complexity There is no agreed definition of complexity. Most definitions start with a minimum requirement that many parts are involved in nonlinear interactions, which create a whole with emergent properties. Other authors insist that the parts have a cognitive model of their role and position in the system but since these cognitive models cannot claim to complete knowledge either of themselves or of the system, the partiality of knowledge is the reason that the characteristic behavior of the whole is seen to be emergent. For this reason, some authors insist that complexity is a property of biological and social systems rather than of physical or chemical systems.

Nonlinearity This occurs if change in one variable does not cause a proportionate constant change in a dependent variable.

Strange attractor A region of phase space, described by a fractal; containing the path of a chaotic system.

Chaos to Complexity

There is a reasonable consensus on what the study of chaotic systems means, at least in formal terms, but there is very little consensus on what the study of complexity entails. This is a generic field, almost a point of view or perspective, a paradigm which conceives of such ordered systems as evolving from the bottom-up. Complex systems consist of very large numbers of elemental units that act almost independently but when examined in their aggregate, display patterns that are highly ordered. In complex systems, order emerges spontaneously from which a hierarchy of functions appears but where such structures are not governed or manufactured from the top-down. Most definitions start with a system composed of many parts which involve nonlinear interactions which create aggregations or "wholes" with emergent properties. These properties are "emergent" because they cannot be predicted from what is known about the parts and their interactions. Sometimes the mantra of the Gestalt movement in psychology is invoked for such complex systems where it said that "the whole is greater than the sum of the parts."

Complexity embraces the dynamics of a system in that its trajectory or path through space and time is defined in such a way that unusual or unexpected transitions can be identified. Complex systems models embrace a range of behaviors from all kinds of chaos or catastrophe to complex equilibria that manifest regular oscillations or cycles. Models of such complex systems can be set up computationally so that we can find out what actually emerges from a known set of interactions, even if we cannot predict the consequences of such complexity in formal terms. Without computers, systems theorists were only able to make small advances in understanding the dynamics and fluctuations of such system behavior. Back in the 1920s, Volterra following Lotka derived equations for the interaction of two species in an ecosystem, using nonlinear functions. The classic example is the relationship between basking sharks as predators and shrimps as prey. If the abundance of shrimps is low, the sharks starve, and there is a population crash. The shrimps can undergo a population explosion, since sharks reproduce slowly. But when the sharks do increase, shrimps decrease at an accelerating rate. The relationships are nonlinear, that is, the rate of change of one variable with respect to the other is not constant. The system results in "repeated" fluctuations in the two species in time (as we illustrate in Fig. 1), which is indicated by the closed loops for different rates of change which show the system in its phase space (see Fig. 2). This behavior is often called a dynamic equilibrium although complex systems models often simulate situations which are far-from-equilibrium in that their behavior never settles down, implying a permanent state of disequilibria.

Although there had been mathematical intimations of this variety of dynamics more than a century ago, it was not until computer simulations began that theorists finally understood there were classes of systems that never went to equilibrium, and never repeated a previous state. One of the first which became a classic example was by the meteorologist Edward Lorenz, who modeled the Earth's climate using three variables linked to each other by nonlinear functions. Fig. 3 shows how these three variables might change through time and this looks somewhat random, but Fig. 4 shows how regular its behavior is when visualized within its phase space. Since the system revisits the same neighborhoods in this space, we can say the path of the system has a kind of shape—here a shape with two lobes like a butterfly. This shape is known as a strange attractor. In this case, the system stays in one neighborhood for a while, then flips to the other neighborhood. The strange attractor actually exists in three dimensions:

☆Professor Graham Chapman passed away on August 31, 2014. Professor Michael Batty edited and added to Chapman's article on **Chaos and Complexity** for the second edition of the Encyclopedia.
†Deceased.

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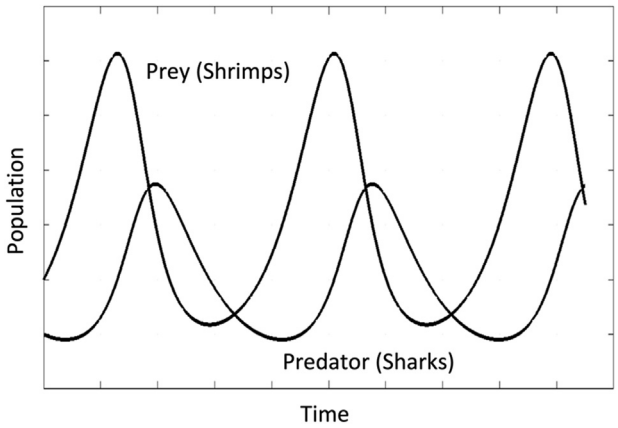


Figure 1 Lagged fluctuations in the populations of shrimps and sharks from the Lotka–Volterra equations.

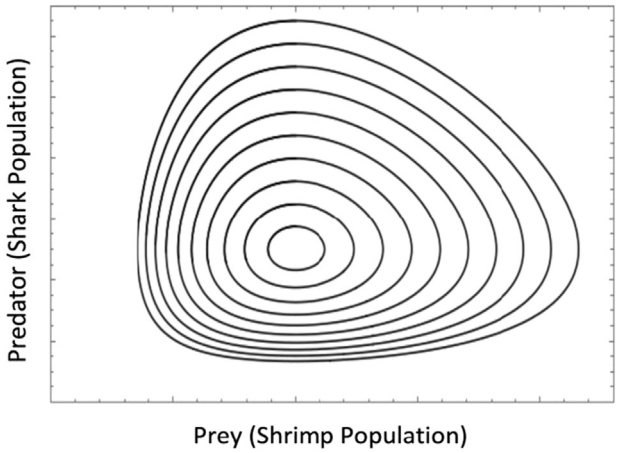


Figure 2 The cycles of shrimp and shark populations from **Fig. 1** represented in a phase space.

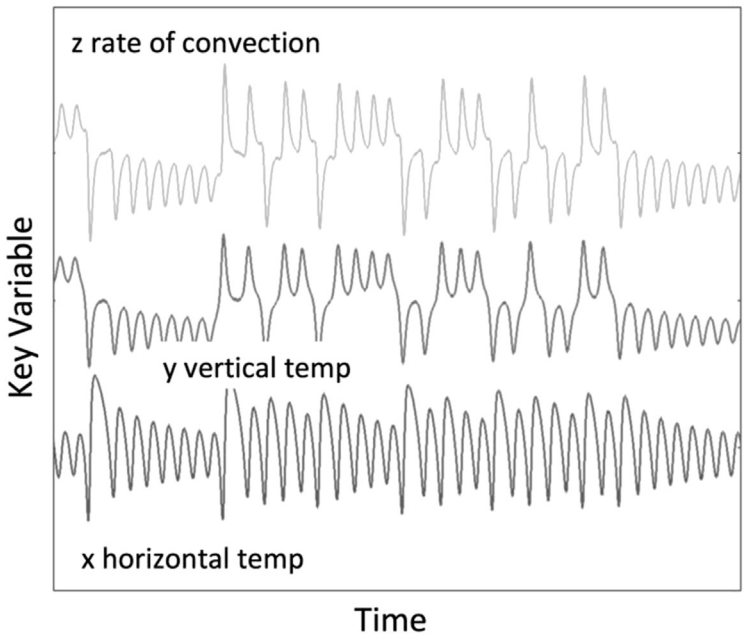


Figure 3 The behavior of a typical chaotic variable in time.

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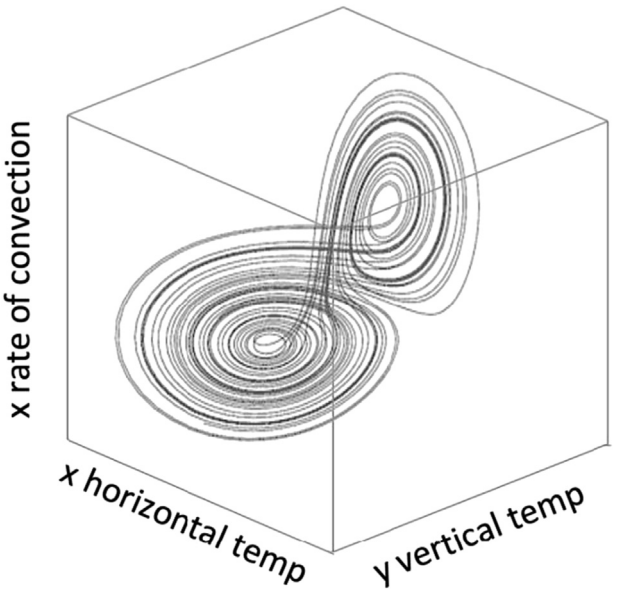


Figure 4 The Lorenz attractor.

one for each of the three variables which are the rate of convection, horizontal temperature variation, and vertical temperature variation. The attractor is shown in Fig. 4 as a three-dimensional trajectory where every positioning of this line defining the trajectory is in a different position in the phase space.

These systems are deterministic in that they have no random component. For any given current state, there is one and only one next state. This means that from any point in the phase space, there can be one and only one line segment going to the next point. Hence, wherever in Fig. 4 it appears that the lines touch and cross each other, they do not. They are separated in the third dimension, and pass either behind or in front of each other. In this sense, although this attractor appears to be a continuous surface, it is in fact a single line.

We can think of the trajectory of some systems as being modeled as a number, where each digit follows deterministically from the digit before. A system that goes to a stable end state might be like 19746538434322222222222222. A system that reaches a dynamic equilibrium may be something like 59137465253414141414141141. With chaotic behavior, the number is essentially random, in that there is no detectable pattern in it, and it never goes to a terminal state. This understanding leads us to the apparently paradoxical term—deterministic randomness. It highlights the fact that systems can be deterministic, but there is no way we can predict the value of the n-th digit, without calculating all preceding $n - 1$ steps. This lack of predictability is a feature of randomness, but not the kind of randomness which originates from flipping a coin. We will return to this feature in a discussion of measures of complexity, below.

So, let us summarize the main characteristics of chaotic behavior we have identified so far: such systems are deterministic, they never repeat themselves, and they manifest strange attractors. Now, consider the two lobes of Fig. 4. Suppose one lobe represents ice age Earth, and the other, thermal period Earth. At some stage in the future, which state will the Earth be in? If the future we are thinking about is tomorrow (one iteration), then there is not much difficulty in saying tomorrow will be much like today. The only way we can find out what it will be like in 300 iterations is to calculate all 300 steps. The future of the system is computationally irreducible. It is not that the system exhibits "counterintuitive behavior"; the behavior has no intuitive characteristics. Further, if we change any of the initial conditions by even the smallest amount, our prediction of tomorrow will change by an almost equally small amount, but the system in 300 iterations may be in a completely different state from the one calculated initially. These systems are infinitely sensitive to initial conditions. This has been paraphrased as the "butterfly effect": that a butterfly flapping its wings in West Texas may or may not set off dynamics that culminate in a hurricane in the Caribbean a month later (notwithstanding the fact that the attractor also looks a little like a butterfly!). In history, the effect has been mythologized as: "for want of a nail, a shoe was lost; for want of a shoe, a horse was lost; for want of a horse, a king was lost, and the kingdom too."

Chaos theory, in fact, is poorly named. It is not at all about what we colloquially call chaos or even about randomness; it is about deterministic systems that are computationally irreducible. Further, it is about the impossibility of ever accurately mapping the real world onto such a system model due to the indeterminacy of the level of resolution. To do so, we would have to have infinitely fine-grained observations of the real world, and an infinitely large model, both of which are impossible. Our increasing theoretical understanding of this problem has made us realize that we will never be able to predict the weather of northern temperate areas for more than a week or two by calculating forward from current system states. The present limitation of most weather forecasts to 5 days is because we are already nearing that limit. This limitation does not preclude some seasonal forecasting based on the historical understanding of repetitive behavior—as with El Nino or the North Atlantic Oscillation—but it does limit predictions that are made for longer than about a week.

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Although at first glance it might seem that we now "know we cannot predict" the future, systems theorists have studied many different kinds of attractors, and to some extent can narrow down the range of possible futures. What chaos theory does is enable us to develop an appropriate perspective on prediction rather than providing us with tools that enable it, and in this sense, it is relevant to both human and physical systems which embrace the range of sciences from the social to the natural and from the artificial to the physical. Chaos thus has wide applicability to a variety of space-time systems of which geographical systems are examples *par excellence*.

Defining Complexity

There may be a degree of agreement on the starting points for the study of complexity but from that point on, complexity becomes a circus big-top for any number of performing intellectual acrobats in an increasing number of circus rings. The behavior of chaotic systems is a-intuitive, not even counter intuitive—that is, we cannot guess what such a system will do, we have to wait and see what such systems do. Complexity is one useful step beyond such an intuitive behavior where the dynamics of the system may well evoke in us some intuitive recognition of holistic behavior. Good examples are when a shoal of fish maintains a coherent pattern, but changes direction to avoid a predator, or when a flock of birds seem to behave as a collective, forming and re-forming a collective V-shape as they fly along their migration routes. In these circumstances, we cannot predict exactly how the shoal or the flock will behave, but if we observe the same species, we begin to see the kinds of behavior that might occur. The same may be said of our understanding of riots at a football ground—we may know more about the pre-conditions that might provoke or prevent a riot, but we are not be able to predict exactly when and in which part of the crowd the riot will start.

Increasing attention is now being paid to networks and emergent behavior. In fact, network science has to some extent become the cornerstone of complexity science. Networks may be richly connected, thinly connected, hierarchically connected, or in any combination thereof. Increasingly we realize that the patterns of connectivity can have significant effects on whether disruptions to the flows on the network links dissipate or propagate. The classic example involves different configurations of road structures and the way their connectivities affect the manner in which jams develop and propagate, as any driver on a high-speed motorway network will realize as congestion builds to thresholds that reflect stops and starts in conditions of saturated flow. Another example is a power grid in a large city which can collapse unpredictably when a minor fault trips a sequence of more significant faults. Put simply, we are only just beginning to develop the tools to understand how complex dynamics develop on different network structures.

The examples of fishes, birds, and humans are intrinsically biological (in that nervous systems are involved) but the example of the power system is physical. Some complexity theorists would claim that these two kinds of cases have the same basis, and that the distinction between biological and physical is an "obsolete disciplinarity" compared to the development of new interdisciplinary ways of thinking that characterize complexity theory. Others would claim that they are radically different and cannot be fused. This dispute lies at the heart of our understanding of what is meant by "science," and what is meant by "being human." In classical science, the future is only an outcome of the present: the only questions that can be asked are "how" questions, not "why" questions. On the other hand, human beings believe they have agency, and can choose a future.

Both camps in complexity theory toy with the idea of "emergence," which implies that when some components are combined, there are properties of the whole that are not properties of the parts, hence the reference back to the Gestalt that we noted in our introduction. At one level, this is trivially true although still mysterious. Two gases, oxygen and hydrogen, combine together to make a compound, water, which has unique properties unlike those of either parent: its solid phase (ice) is less dense than its liquid phase, and hence ice floats. At other levels, it is nontrivially true. Time and again one can read that the most complex entity in the known universe is the human brain, and human beings have self-consciousness. Where does this self-consciousness come from? The only answer so far (if one accepts the question as being meaningful) is that when a sufficient number of neurons are connected together (and it is a number in the billions) consciousness "emerges" as an effect that is not attributable to the properties of the neurons and synapses of the brain. That is to say, given enough quantity, qualitative changes occur.

Those who claim that the shoal of fish and the power system can belong to the same class of complexity are basing their claim on this idea that complexity emerges as a result of quantities—of elements and of connections between elements. The problem is that with consciousness comes agency. For "humanists," choosing a future implies that humans can imagine a model of a future, and that the choices are purposive and nonrandom. The question that we cannot answer is: "How do you explain behavior simultaneously as an 'outcome' and as an 'intention'?"

Modeling Complexity: Cellular Automata

The most generic model of complexity which encapsulates emergence in space and time is called a cellular automaton (CA). These models were first proposed during the Manhattan Project at Los Alamos during World War II by Stanislaw Ulam and they were then picked up by John von Neumann, who argued that they represented a simple analogue for how growth processes could be modeled on the newly invented digital computer. They were first articulated as temporal processes on a two-dimensional lattice, which enabled changes in the state of the cells defined by the lattice to be made through concentration, diffusion, transitions of various sorts, and local action at a distance, all relating to the cell in question and its neighbors. CAs have thus become ubiquitous in

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exploring many aspects of what is claimed to be emergent behavior. In their simplest form, they are large, two-dimensional "boards" of many squares arranged in a rectangular grid. The cells can take on as many states as relevant to the problem but in their simplest form the states are developed or not developed—being colored, say, black or white. Any set of initial conditions of black and white may be used and, in any given simulation run, a set of transition rules is used to determine how a square may or may not change from black to white, or vice versa, in any one iteration.

If the corner (diagonal) neighboring squares are counted, any square can have eight neighbors. For all unique combinations of black/white states for these nine cells, a transition rule specifies what the state of the central cell will be after the current iteration. There are therefore $2^9 = 512$ black and white combinations of the nine squares overall that will give an output state of either black or white—developed or not developed—for the target square. For many starting states and for many transition rules, patterns simply go to some stable and possibly monotone state; however, there are examples where dynamic patterns emerge which seem to behave "organically."

One of the best exemplars is John Conway's famous "Game of Life," invented in 1970 as a board game in which a change in the state of any cell can reflect a birth, a death, or no change, the cell changing to black, white, or remaining the same color (see Fig. 5). The reader is urged to visit the website at the end of this article to explore this game from which they can experiment to generate many of the "life forms" which emerge from the interactions of neighbors on the grid. The neurophilosopher Daniel Dennett used this game as a starting point for his attempt to show that there could be a solution to the "outcome/intention" paradox in a book which has a similarly contradictory title: *Freedom Evolves*. Just as systems that go to equilibrium or dynamic equilibrium are of little interest to the chaos theorist, so also the complexity theorist is not interested in those simulations which reach a static terminal state.

By contrast, other CAs have many significant and interesting properties. They are computationally irreducible—only by running the CA 1000 iterations can you know what state it will be in after 1000 iterations. Each cell is a "self" with a boundary, and it has "knowledge" of its own state and that of its immediate neighbors—so it is in an environment with limited spatial knowledge that is not total knowledge. Its future states depend on how it reacts to this knowledge. Dynamic patterns self-organize and higher scale (multicell) entities emerge, which have characteristic behavioral patterns. Indeed this allure is so strong that there is an international "Game of Life" club with its own vocabulary describing changes in state using the terminology of "guns," "breeders," and "gliders"

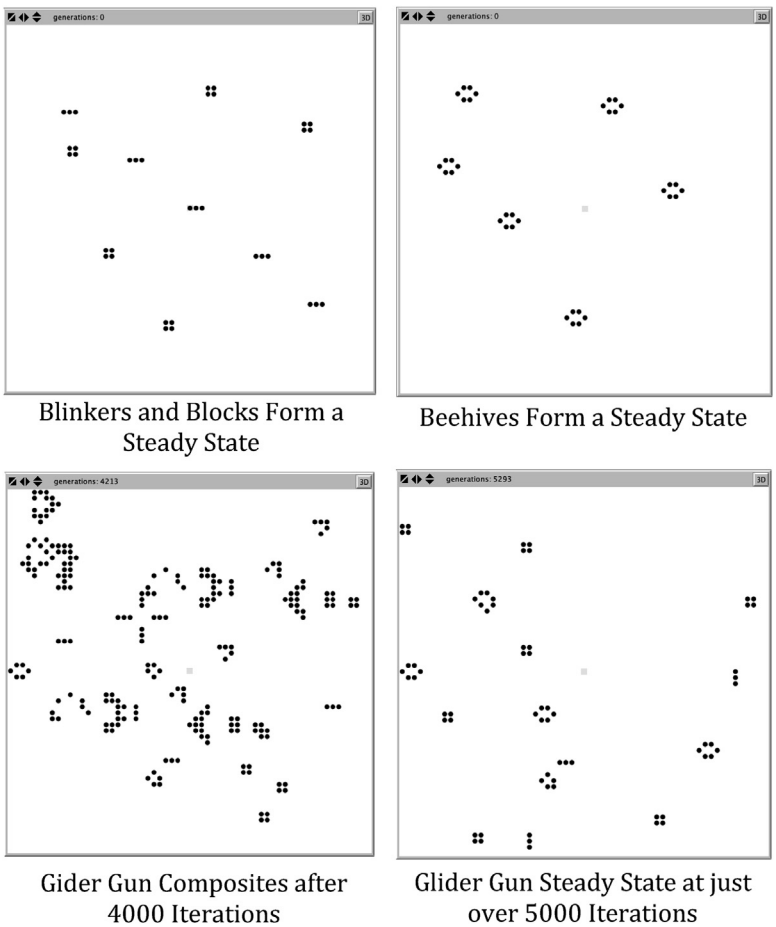


Figure 5 Breeders, guns, and gliders in Conway's **Game of Life**.

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that they have coined for different sets of transition rules. "Guns" are static shapes that fire "projectiles" at regular intervals in straight directions; "breeders" are shapes that produce new versions of themselves; "gliders" are shapes that maintain their shape as they drift across the board. It is the iteration of these rules that generates the patterns, examples of which we show in Fig. 5.

This seems like a perfect metaphor for emergence, but actually it is not. The behavior of the whole is explicable by the behavior of the parts. Although it may appear that a "breeder" is an organic whole, it has no self, no boundary to self, and no central knowledge processing. All of that still occurs at the level of the individual cells. The "organisms" do not evolve—they might get eaten up or disintegrate, but the stable shapes that wander round the board do not randomly mutate and form new species. The simulations might, however, be good metaphors for the shoals of fish—there is no central processing there either, although an onlooker can sometimes get the sense that there could be. The behavior of the whole shoal may affect the survival of the species of fish, but this is an outcome, not an intention.

It is the human observer who imbues the fascinating patterns of Conway's "game" with the dignity of "life." These "creatures" lead frictionless lives in which they consume no negentropy, nor do they produce entropy. Notwithstanding this criticism, enthusiasts ardently pursue the field of artificial life of which this simulation is a founding cornerstone. There is no doubt considerable value in using these models to throw light on some kinds of complex dynamics. A favorite example is modeling traffic flows on motorways, where traffic can accelerate, then decelerate, and bunch up, often forming standing waves at points where there may have been an incident hours earlier. Running a model in which simple rules relate to how cars adjust to each other in terms of spacing and speed can mimic these effects. In fact it is the randomness of how drivers react and speed up that often causes the waves without any particular incident as such.

The Big-Top of Complexity

Many of the topics that now vie for attention within the complexity circus had their origin somewhat outside this new field, or even before the new tent was erected. Examples include artificial intelligence (AI) which itself comes in a variety of incommensurate forms. These include expert systems, which quarry human experts such as doctors diagnosing a patient, to elicit the rules underlying their decision-making. Others may depend on artificial neural nets (ANNs) which enable patterns to be discovered in big data explicitly using a model of data processing in analogy to the way neurons are related in the brain. Neurons are connected in nets, and "fire" each other off, or not, according to specified thresholds. They "learn" to replicate the same relationships between their input data and outcomes they are designed to reproduce. The problem with the latter is that the relationships that link data with output cannot be isolated from the network: they are "black boxes," and in this sense, the patterns that are generated are truly emergent but often not explicable. It is then difficult to build confidence that the network will correctly identify novel cases. An apocryphal anecdote relates that the US Army trained a neural net to distinguish between Soviet and US tanks by inputting thousands of photographs of each to the net. When it came to using the net in the field to distinguish at high-speed between friendly and enemy tanks, all the tanks were found to be friendly. It turned out that the net had originally distinguished the two types of tanks in terms of whether the photography was clear (a US tank) or grainy (a surreptitious distant shot smuggled out of the Eastern Bloc). All real-time images were clear; therefore all tanks were friendly. Notwithstanding these limitations, ANNs can still be useful and are now the basis of machine learning and deep mining. They have been quite widely used in spatial analysis in a variety of ways, to examine patterns in spatial interaction through to classifying ecosystems from remotely sensed satellite imagery but the verdict is still out on how good such methods will be at dealing with complex systems in the long term.

A similar "blind" and "self-organizing" tool is genetic programming, where thousands of arbitrary rules are assigned to solve a task, and those that are most successful in any generation are selected for the next iteration. The programs often involve random mating (swapping elements of code) so that the fitness of the genes can be established and an optimum configuration of the problem in hand be generated. Another popular development within the big-top is the use of agent-based models (ABMs). These are in principle rather more sophisticated versions of CAs, cells on which agents or objects can move. An application within geography is to the modeling of cities, seeing how social segregation, or suburban sprawl, may result from interacting agents with differing behavioral rules. Studying the behavior that emerges, and testing new hypotheses and new rules, can help understand urban processes in a way which is radically different from old-style classical spatial economics. These models can also be operationalized by linking them to GIS systems so that the "board" becomes the "real" city.

Developments in robotics may also come within the circus tent, not so much for applications within industry, but for a better understanding of biomechanical principles. The construction of a mechanical dog that can run with the same gait as a real dog reveals that only some of the simulation needs to be in information processing. Much of the performance of the robot is better built into its own mechanical structures and material balances and tensions, so that the control is stimulating "inherent response" harmonics of bouncing and stretching. Perhaps the most far-reaching of complexity's current buzz-words is the idea of the "tipping point." The implication that there are thresholds beyond which stability becomes instability, or leads to an irreversible, alternative, and undesirable state, has been given political legs, and is used almost regardless of whether there is a well-founded model behind the tipping point or not. So in that sense, complexity has entered our imagination as a somewhat frightening, certainly mysterious metaphor in terms of the following typical clichés: the climate will soon pass a tipping point, immigration will soon pass a tipping point, and the educational system will soon pass a tipping point. We need to be careful about our use of this new science in the popular imagination.

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Measuring Complexity

There is little doubt that many of the systems that we as geographers are concerned with from both the human and physical perspectives are getting more complex as they evolve. Life itself is the obvious manifestation of this increasing complexity and, by this account, it ought to be possible to provide a quantitative measure of complexity, so that the extent to which "this system" is more complex than "that" can be expressed quantitatively as well as qualitatively. Many authors believe that no measure of complexity has yet been discovered that has any useful meaning. However, there are some measures which attract intellectual curiosity, and perhaps in the future some significance will be derived from them. The most cited is the Solomonoff–Chaitin measure of algorithmic complexity, which says that the degree of complexity is defined by the shortest possible algorithm that can express the observed behavior. Formally this is defined in terms of the length of a program in a universal Turing machine (the most basic theoretical computer) required to generate or print any given output.

To return to our numerical examples, a single decimal digit like 4 or 6 requires 3.322 bits of information for it to be measurable. Suppose the behavior of some system is represented by the string 0.33333333333333333333… which is 100 decimal numbers long. To express 100 such numbers requires 332.2 bits. But we can also express the number as $n = 1/3$. This expression requires only 2×3.332 bits, plus a small length of program for calculating the division. The number is clearly much less complex. By contrast, a string of 100 numbers from a chaotic system cannot be summarized by an algorithm that is shorter than the 100 numbers, and it has therefore "complete" complexity. Some fairly simple reasoning leads to the conclusion that the proportion of numbers that are not complex becomes vanishingly small, implying that stable solutions are extremely rare.

In statistical terms, if some sample of numbers are randomly drawn from a normally distributed population, again we need not know all the numbers; we can use an algorithm to specify the distribution in terms of a few parameters such as the mean and standard deviation. We need not know all the details of each number. Apparently complexity has been reduced. The use of statistical analysis in social science has thus been based on the idea that it is the aggregates, not the individuals, which matter. However, no two nontrivially sized samples will contain the same numbers (hence the idea of the standard error of the sample mean).

Now, suppose the behavior of the whole is not an aggregate, but the emergent effect of individuals interacting with each other. Then, clearly, the initial conditions in the two cases are different, even if the sample characteristics are the same. So if we are interested in emergent dynamics, the statistical approach will not work. As an example, suppose people have an individual and static propensity to riot. Then a statistical analysis of a sample of football fans will work out quite easily the likelihood of a riot of intensity x; however, if the propensity of each individual to riot is changed by the behavior of a neighbor (and worst of all it is a positive feedback loop), then what matters most is being able to predict a single trigger point somewhere. Such a trigger point may be the simple adjacency of two people who have poor self-control and competing loyalties. Then there are further adjacencies to consider—whether those surrounding the flash-point pair are likely to suppress or aggravate the situation. Even the same crowd of people may yield completely different behaviors depending on who is adjacent, or who is connected to whom.

There is really only one possible conclusion about the state of our knowledge of our own societies. They are intrinsically complex, and all models of them introduce simplifications that render model predictions unreliable. This is one of the pessimistic conclusions that complexity brings to the party, that the world is unpredictable, and that no model can ever reproduce reality in a way that captures the essence of any complex system. In fact, in geographical systems, particularly those dealing with human systems, complexity is always increasing while entropy is always decreasing and this poses limits on our understanding that in turn limits our powers of explanation, and of course prediction.

Properties of Complex Systems: Scaling and Fractals

Although our focus on chaos and complexity here has been dominated by limits on what we can explain, model, and predict, there are properties of complex systems that are generic, perhaps even universal, that enable us to make comparisons between different applications. We have not said very much about the size of such systems, although we have implied that complexity does not relate to size *per se* but to more qualitative features such as the processes of evolution that give rise to emergence.

Complex systems, however, do manifest regularities with respect to their size and the distribution of their elements. Many such systems are built around the idea of their elements competing with one another for limited resources. Take cities, for example. Cities grow up where people come together to pool their resources which is the economic imperative based on increasing prosperity and opportunity. In this sense, there are many cities as most of us cannot live and work and exchange our resources in the same place and in any case, physical resources that are required are not evenly distributed. Cities thus compete with each other and over time, all of them cannot grow to the same size. It is obvious that there will be a much smaller number of large cities than small cities. So as an exemplar, the system of cities represents a generic class of objects or elements—individual cities—that become ordered in some way as they evolve.

The distribution of such elements is extremely well defined and follows what are called power laws. In other words, when we look at the size of cities by their rank-order, we find that their sizes fall off as a nonlinear relation that follows an inverse power law, which in its purest form suggests that the size of any city at any rank in the set is the size of the largest city in the set divided by its rank. There are other ways of saying the same but, in essence, many complex systems show a distribution of their elements that

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follows this rank-size rule. In the social sciences, the distribution of incomes, the distribution of talents of many kinds, the distribution of words in a novel, and so on all follow the same kinds of power law. In the physical world too, the distribution of many geomorphological objects follow the same laws: the distribution of any phenomenon that involves energy discharge such as volcanic eruptions, earthquakes, moon craters, river flows, and so on.

This observation in and of itself is not particularly insightful for it has been known for many years; but it is what it implies that is important. It is an example of fractality in that when we examine a power law, it is clear that the relationship between, let us say, size and rank is scaling in that when we take a piece of the distribution and scale it up or down, we find the same distribution. This is the property of self-similarity. It means that wherever we are on the distribution, whichever way we look, up or down, we see the same function. In short, if we did not know what city size we were looking at, just having acknowledge of the relationship would not tell us which city because at that point in the distribution, size is not relevant; it is the relative distribution that is. For example, if our city size could be predicted from the rank-size power law as $P(r) = Kr^{-1}$ where $P(r)$ is the population P of the city at rank r and K is a constant of proportionality, if we were to scale the rank by s as sr, the new population would be $P(sr) = K(sr)^{-1} = s^{-1}Kr^{-1} = s^{-1}P(r)$; this is a simple scaling which implies the fact that the distribution of city sizes is self-similar.

We do not have time to explore this here but we can generalize these kinds of relationships to space and time in that we can demonstrate that the way cities, rivers, craters, and so on are distributed are fractal-like in space and time, in terms of their configuration and packing in Euclidean space and their clustering in time. In terms of complexity theory, however, we are but at the beginning in terms of mapping out a new paradigm which will provide us with a template for dealing with complex systems in future research. These are brave words indeed for they imply very different ways in which we need to think about human, natural, and physical systems but remember that it was Stephen Hawking who in speculating on the most important advances in future science in the year 2000 said that: "I think the next century will be the century of complexity."

Conclusion

Systems that are perfectly organized (ordered) do not evolve unless by outside intervention. Systems composed of individuals which exhibit purely random behavior, like molecules of gas in a cylinder, do not evolve, and are best described in terms of their aggregate properties. Poised between randomness and order are other systems. Of those that are biological, we can state that we, the observers, think they have the following properties. They have a large number of elements because when the number of elements increases, formal description of the relations between elements ceases to give an understanding of the whole. There are dynamic interactions among this large number of elements, and many of these interactions are nonlinear. There is feedback within the system. They have a history and every element of the system is ignorant of the behavior of the whole. With regard to complex reflexive social systems, none of the mappings between these conditions and the "real world" is unambiguous. In short, the systems that are candidates for complexity evolve and are intrinsically unpredictable: cities, the atmosphere, rivers, economies, the list of systems with which we deal as geographers is endless, and slowly but surely being informed by the science of complexity.

With reference to human geography, complexity has made an impact as a metaphor, the value of which Thrift (1999) has compared with actor–network theory—where again there are many parts not fully aware of the way their behavior is mediated by the network. Harris (2007) has used insights drawn from complexity theory to grapple with the interface between society and planet in the quest for sustainable development. In more applied terms, ABMs are used to study system evolution, for example, with regard to cities by Allen (1997) and Batty (2007), and these relate strongly to the dynamics of their networks. These models may provide us with many insights into how systems can behave, and how they might behave, but very little by way of predicting their exact behavior. The most certain prediction is that they will be unpredictable because of sensitivity to initial conditions and bifurcation points.

See Also: Cellular Automata; Fractal Analysis; Neural Networks.

Further Reading

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Author's personal copy

Relevant Websites

By far the most relevant web site is that developed by the Santa Fe Institute as *The Complexity Explorer*: see https://www.complexityexplorer.org.
The Complexity Explorer provides online courses and educational materials about complex systems. Complexity Explorer is an education project of the Santa Fe Institute - the world headquarters for the study of complex systems.
The *Lorenz equations* are at https://www.complexityexplorer.org/courses/94-nonlinear-dynamics-mathematical-and-computational-approaches/segments/7975.
And the *Game of Life* is at www.complexityexplorer.org/explore/glossary/258-game-of-life https://www.complexityexplorer.org/explore/virtual-laboratory/126-conway-s-game-of-life.