School of Geographical Sciences and Urban Planning
Arizona State University

SPARC
SCALE WORKSHOP

A Hierarchy of Cities

Qualitative Transformations with Scale

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An Outline

- Scale: underpinning transformations across scales
- Qualitative and quantitative change
- Defining spatial objects: hierarchies of cities, regions and nations: what is a city?
- Breaking up Britain at different scales: fracturing Britain: the politics of recent history
- Defining cities at different scales using percolation theory
- Income and city size in Britain
- Extensions: Europe, integration and fragmentation
- Smaller scales: inside the city

Scale: underpinning transformations across scales

- A good deal of spatial analysis assumes we are looking for distinct patterns that manifest themselves at different 'spatial' scales
- This can be reflected as variations across scales or as invariance at different scales, such as in fractal geometries
- There is very little established theory as to what we might expect as we move across scales, except perhaps rather loosely as for example in fractals where a stable process might scale and manifest itself in similar geometric patterns as in self similar growth – in tree-like structures and such like

- Even with such a nice idea as fractals, it is hard to identify self-similar processes, despite some beautiful graphics that appear to definitively establish order and emergence.
- In fact, a lot of work has gone into thinking about how to relax fractal order and to consider multiple processes in different spatial levels and regimes – the idea of multifractals has become significant

PHYSICAL REVIEW E **92**, 062130 (2015)

Multifractal to monofractal evolution of the London street network

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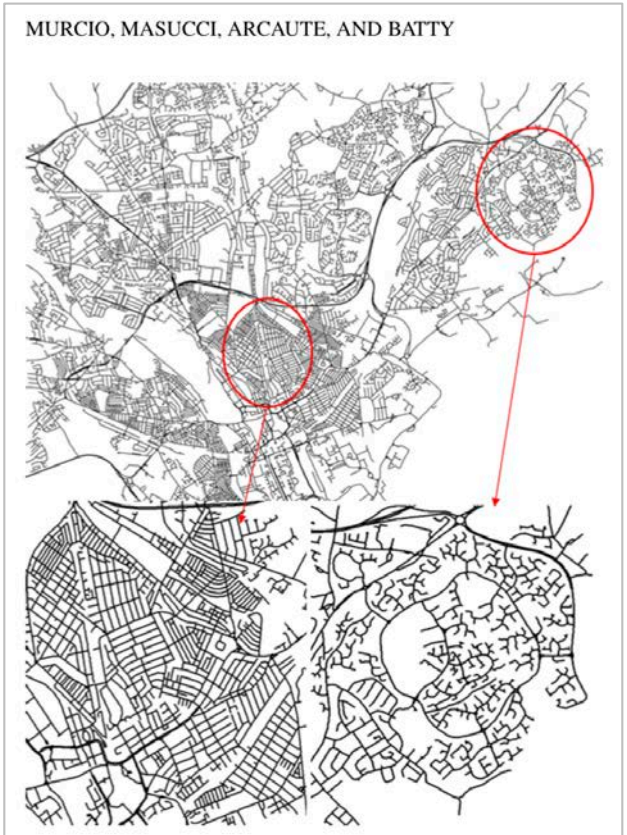


FIG. 1. (Color online) The street network of Cardiff, UK, along with a zoom of two particular areas.

- Although not particularly popular in spatial analysis, there is another approach that seeks to determine what sort of variations there are at different scales prior to any such analysis.
- In other words, theory sometimes dictates what actually happens at different scales and sometimes there are clear transformations between scales. Historically we can see different cultures reflected in this – the previous diagram shows this
- But somewhat more appealing theoretically is the notion that as a phenomena changes – grows for example, there are subtle changes in the way they manifest themselves at different scales – cities are the example par excellence.

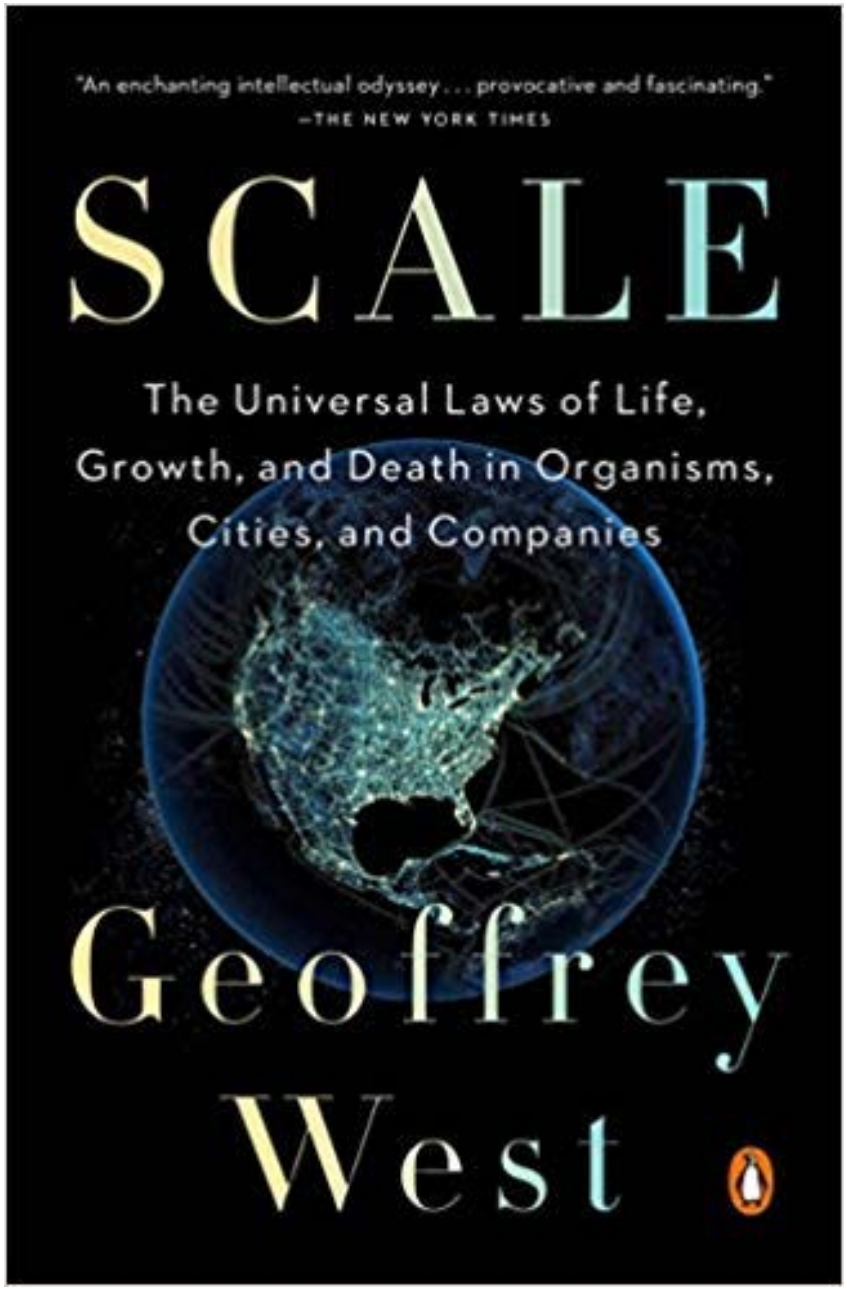
Qualitative and quantitative change

- Let me begin by noting there are rather strong theories about urban agglomeration economies – economies of scale – which kick in as cities grow and become denser.
- In recent thinking, these scale economies might be measured and tested by examining how income changes with population size and if income Y in cities does grow more than proportionately wrt to city size P, we say there is super-linear scaling. The following model might be tested

$$Y \sim P^{\alpha}$$

$$if\ \alpha > 1\ then\ it\ is\ best\ to\ live\ in\ bigger\ towns, you\ get\ more\ money$$
$$if\ \alpha < 1\ then\ it\ is\ best\ to\ live\ in\ smaller\ towns, you\ get\ more\ money$$

- There are many variants of this model and not a little controversy about whether or not big cities generate economies of scale or diseconomies
- Or rather do economies outweigh diseconomies?
- Geoff West has written a whole book about this and it comes under the rubric of allometry. Much was developed in the 1920s in my college UCL



- Let me in passing throw some variants of this scaling model onto the table – basically what this model does not do is make a distinction between poor and rich people in cities. You could have a situation where the poor are increasing and get lesser income per head in bigger cities while the rich become super rich wiping out the losses of the poor while the proportion of poor are increasing
- Our model might divide the population into equal numbers of rich and poor

$$Y_{poor} \sim p^{\gamma} \; and \; Y_{rich} \sim p^{\theta} \; with \; \gamma < 1 \; and \; \theta > 1$$

$$Y \sim \left(p^{\gamma} + p^{\theta}\right) \sim P^{\alpha} \;\; where \; \alpha > 1$$

- So one has to have some theory to test this hypothesis – and this means that strictly we need income distributions – something that we find very hard to get for the town/city level. We can't get it in the UK as we don't ask the income question in the Census
- The other big question mark over all of this involves 'how we define a city'. This is an age-old question. I could spend a lifetime telling you about this but in essence the measures we get of a city are always ambiguous. Some cities have all the poor people on the edge, some all the poor in the centre; some have all the rich salting away their money in cities where they do not live, some do not register the poor or the migrants and so on …..

Defining spatial objects: hierarchies of cities, regions and nations: what is a city?

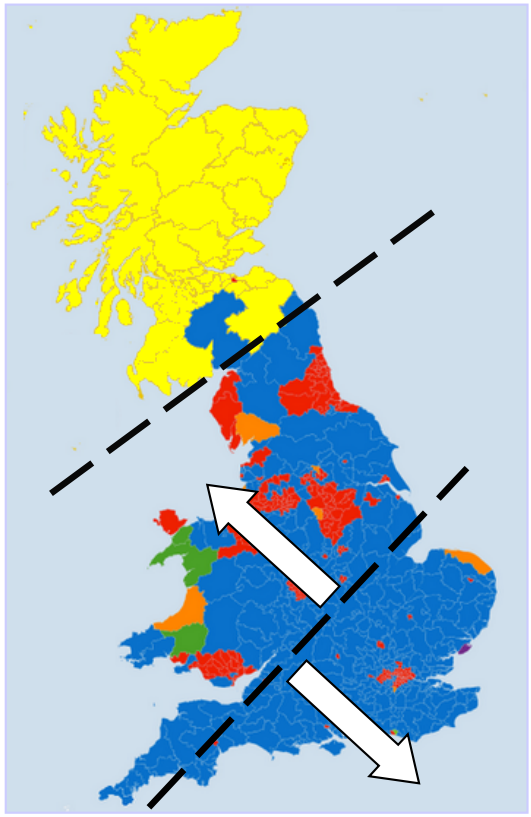
- Now what we want to do here is to look at the different scales of cities – different sizes – and then figure out if the crude formula has any bearing. Its has been tested several times already and in general more or less bears out what West et al. are saying.
- But if we vary the definition of the cities in the set, then the conclusions can be the opposite; we can get sublinear scaling or as we suspect in Britain because it is so hard to define cities simply linear scaling – where it doesn't matter what town you live in when it comes to income.

- There are other issue. As cities get bigger they change in scale – size and scale are related – so how can we test for the effect of scale when the objects themselves are defined by their scale?
- Moreover, big cities embrace little cities in their hinterlands making them hard to define – do we include them but we soon go into infinite regress– where does a city begin and where does it end
- In Britain for example, it is said there is really one big city – London plus the rest, and it gets worse when you get into the Randstad and Belgium.
- And so it goes on – this is a conundrum that rarely anyone has addressed – if the objects depend on scale, how can we use scale to explain them?

Breaking up Britain at different scales: fracturing Britain: the politics of recent history

- Let me digress a bit because on my way to defining cities we need to define nations and regions and scale in Britain is so culturally bound.

As many of you know we have a hierarchy of countries and regions in the UK – this example is for Great Britain - Scotland, Wales, England,



General Election 2016

- Essentially to define cities we are able to perform a regionalisation of Britain using the street pattern and network that contains all the cultural and social history of the last 2000 years.
- Let me emphasise this point about the street-road network which contains a lot of the economic social and cultural history of Britain, from the Celts, to Romans, Anglo-Saxons, to Normans and so on







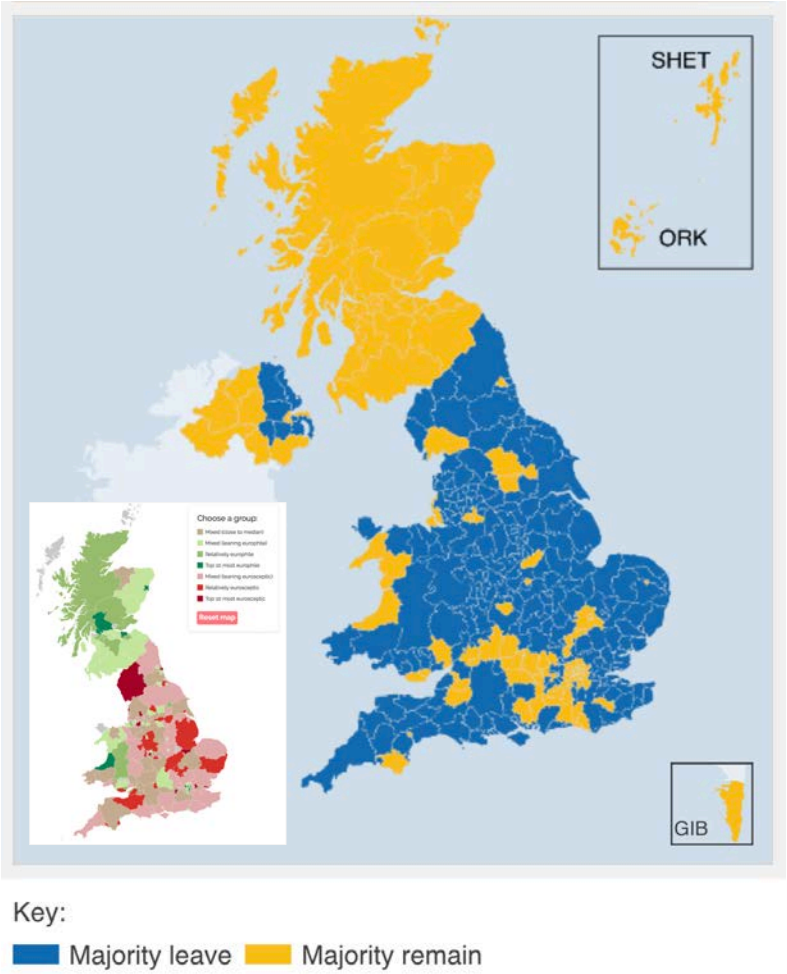




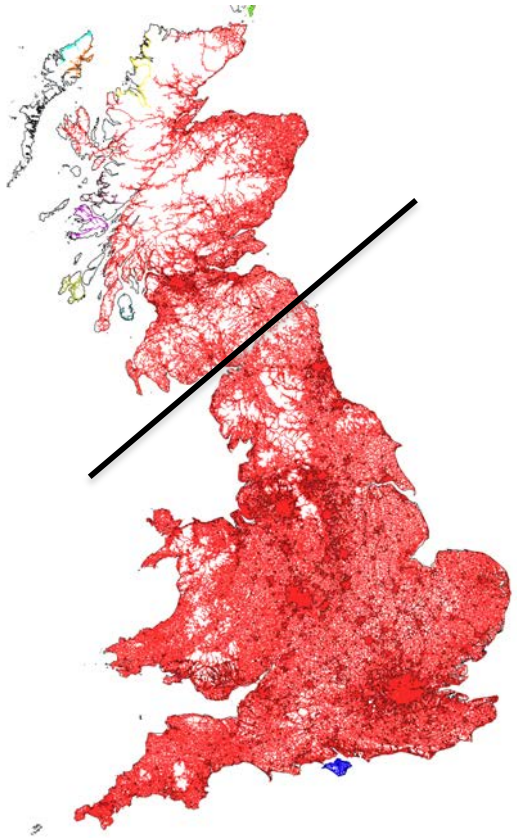
Defining cities at different scales using percolation theory

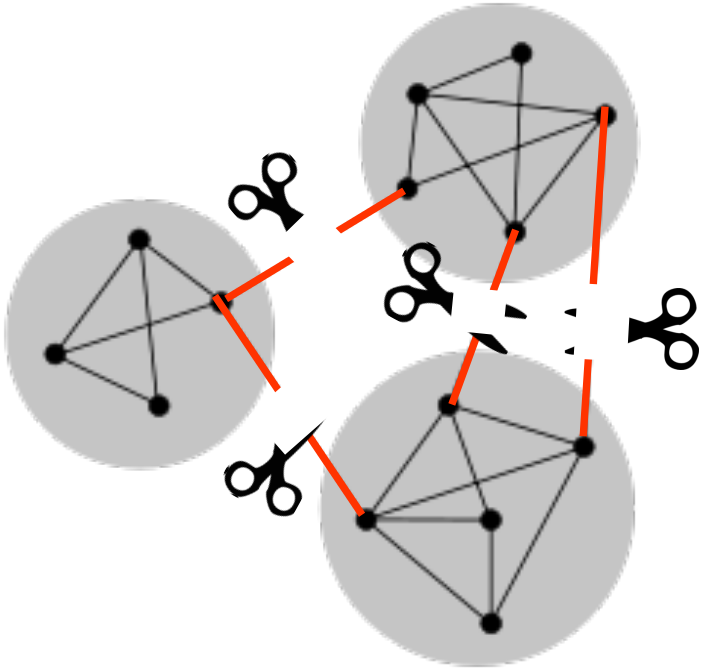
I need to update you with recent events when we British voted to leave the EU.

Let me show you the map of who voted to leave the EU and who voted to stay. I will let you draw your own conclusions but fracturing of the kind I have been showing it key and so it the city-country difference



So how do we fracture Britain to predict its nations, regions and cities? Imagine the network of streets and roads which covers these entire islands. Let us cut the longest links – those that are greater than a distance threshold – so that we partition the network into clusters, and we keep on doing this …. by tightening the threshold, thus building the hierarchy top down

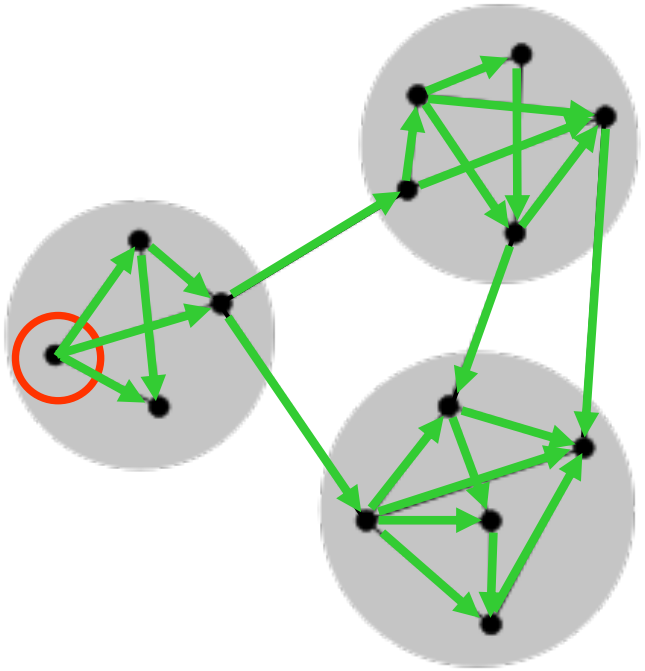




I had given variants of this talk a few times and then someone asked me after the lecture ***"Just exactly what is percolation?"***. So I had better tell you a little more explicitly

Imagine a process that diffuses from a source one step at a time. If the diffusion takes place on a network, as the diffusion continues, it takes in more and more of the network. At each stage more and more clusters are embraced and the biggest cluster grows in size – this is the giant cluster. Eventually all the network is covered. Think of a forest fire. Here is the little network example again.

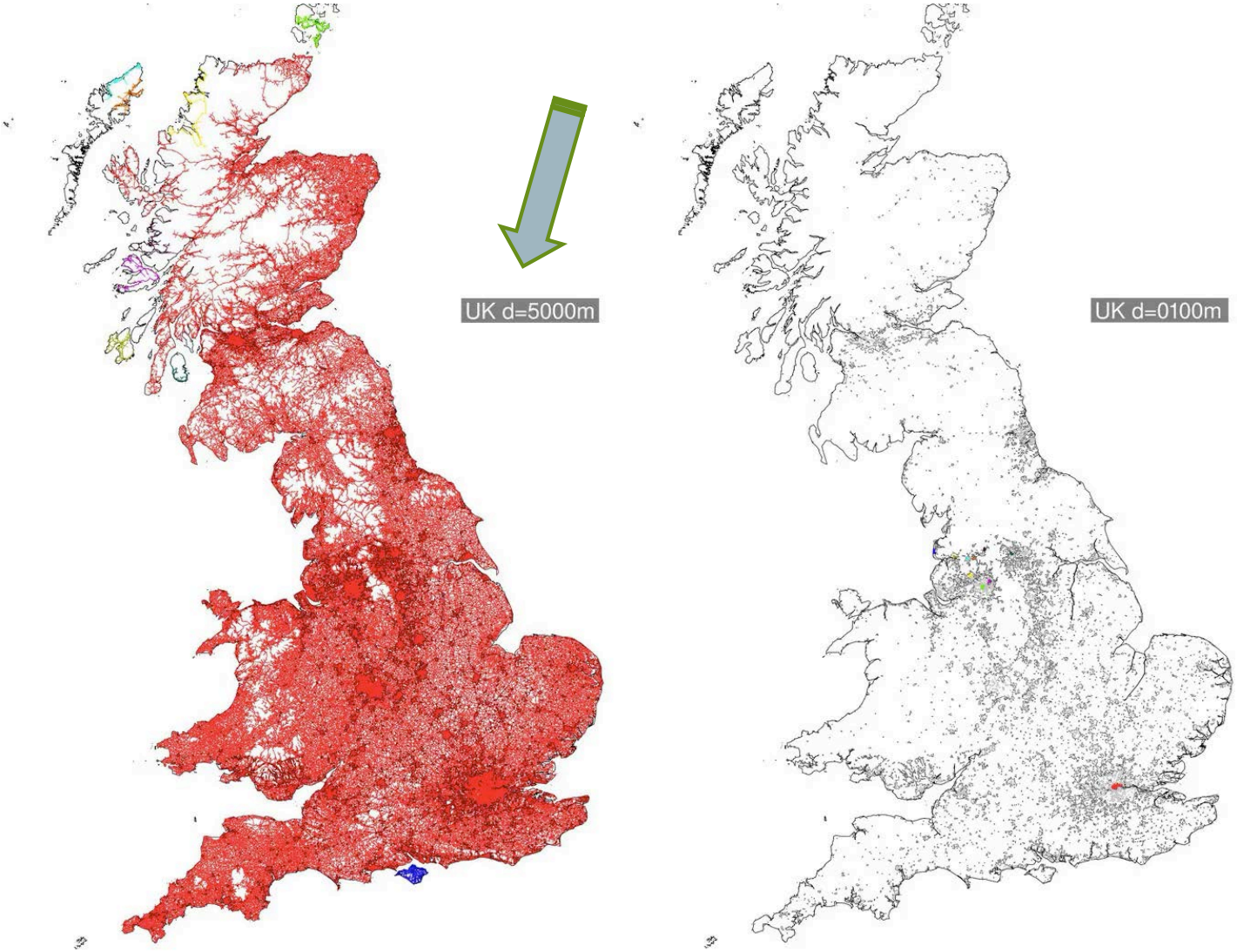
Source



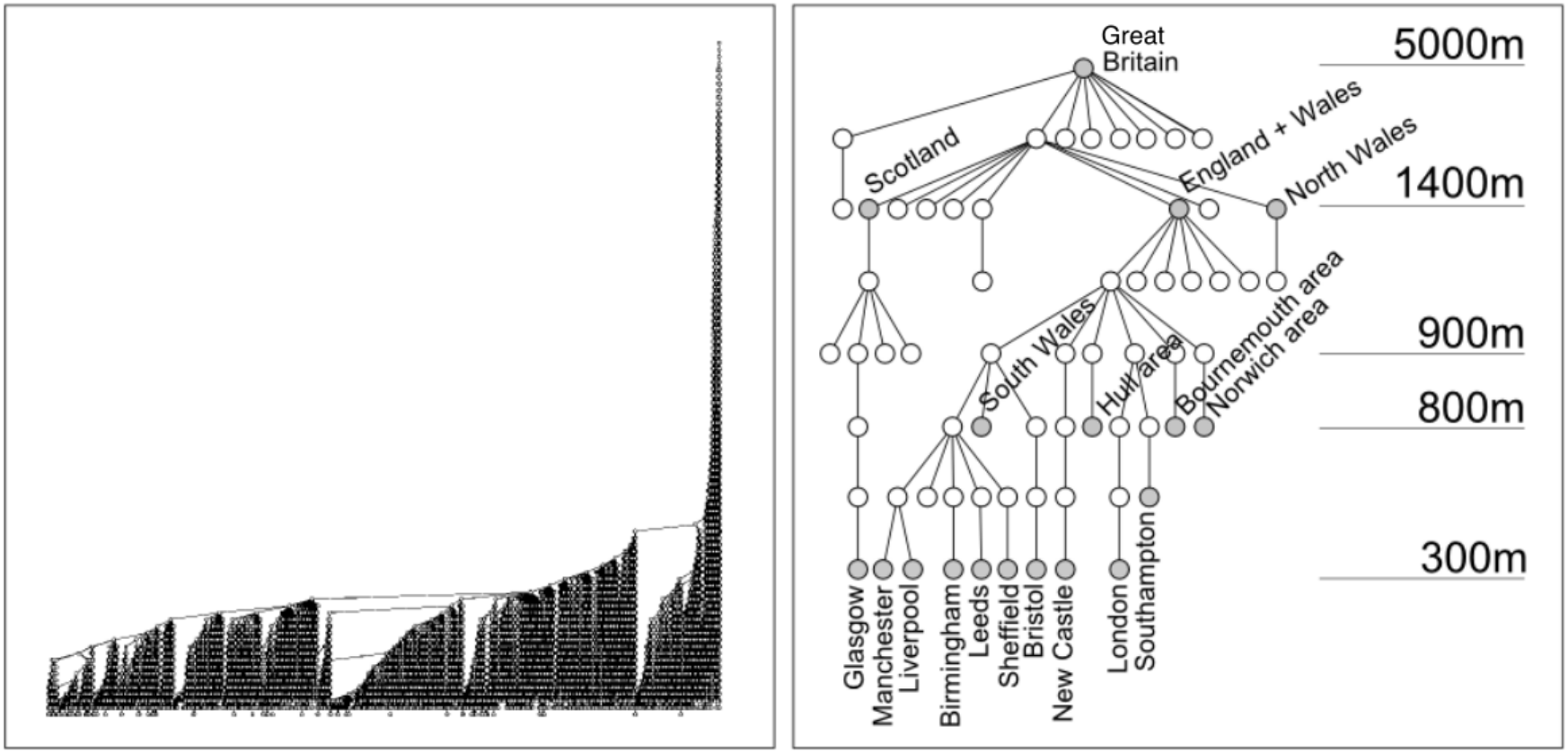
If you play this forwards we build clusters from the bottom up and backwards we build clusters from the top down

We start at 5km and then we progressively build the clusters from the bottom up which assembles the hierarchy from the top down.

This produces nations, regions and then cities which is our main focus, note the Scottish division which I will return to at the end



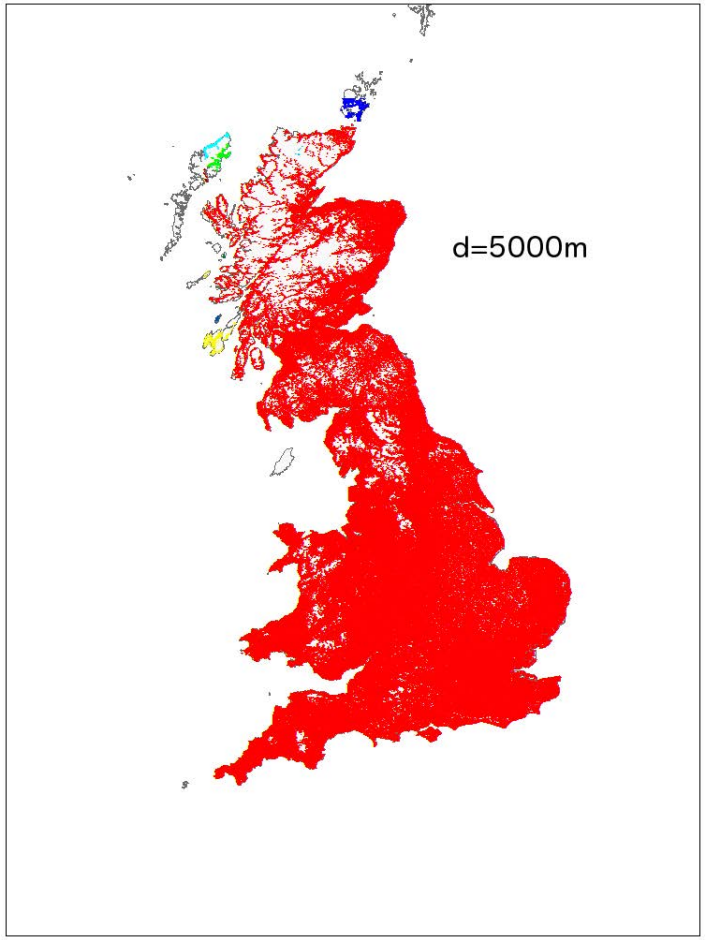
We build or extract a very detailed hierarchy with tens of thousands of clusters



Let us make it a little more relaxed and look at the key stages – the essential issue from the election and what people vote is that *Britain breaks up into its historical cultural pieces*.

Fractured Britain, probably ***Fractal*** Britain

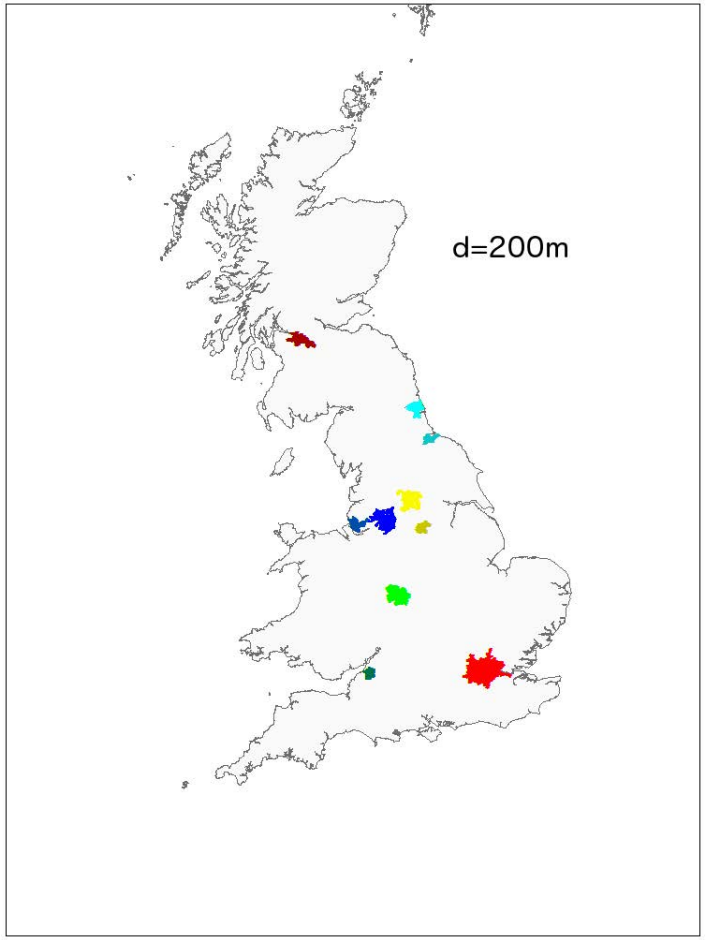
The movie shows it beautifully but so do these frames …..



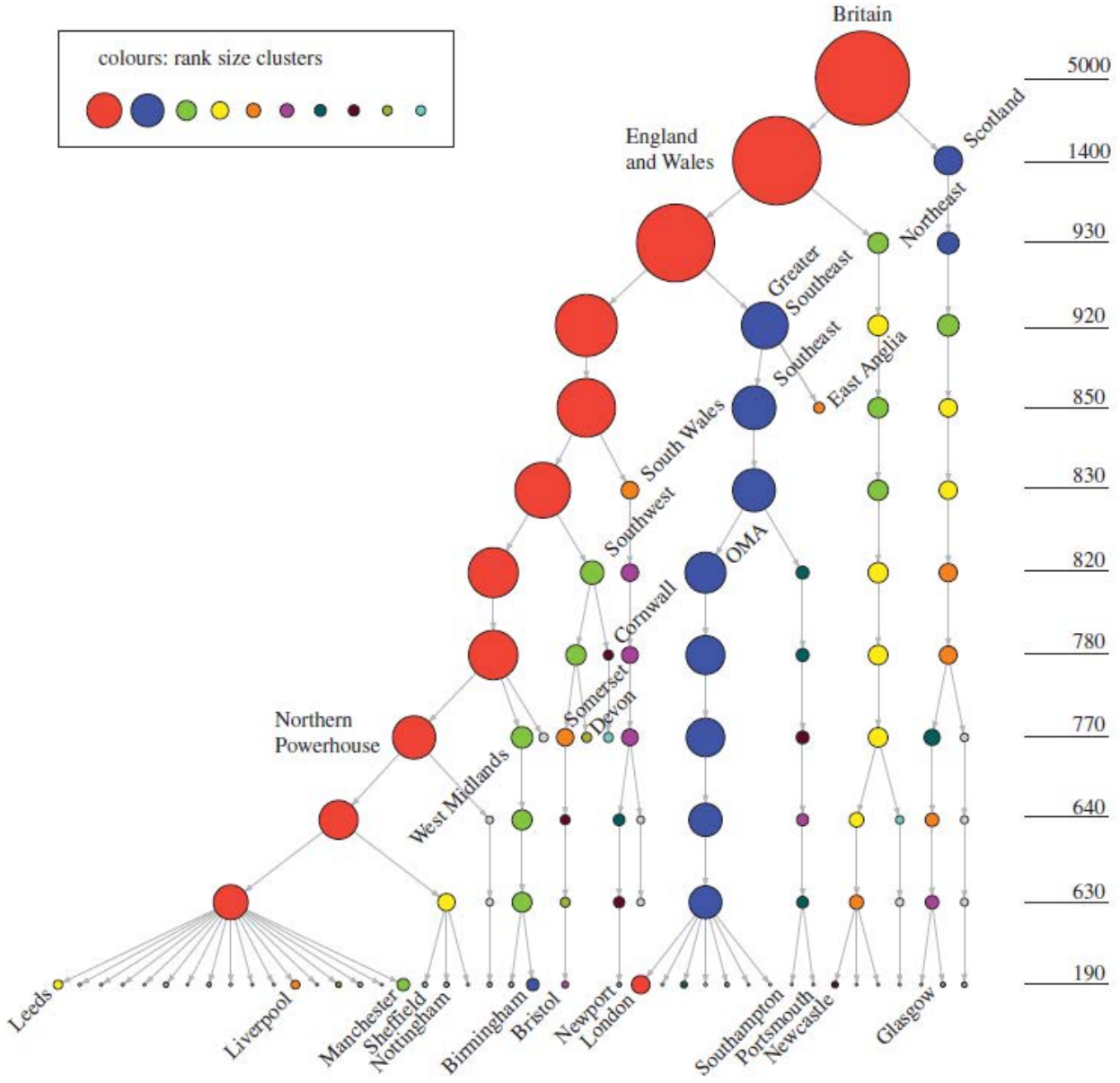
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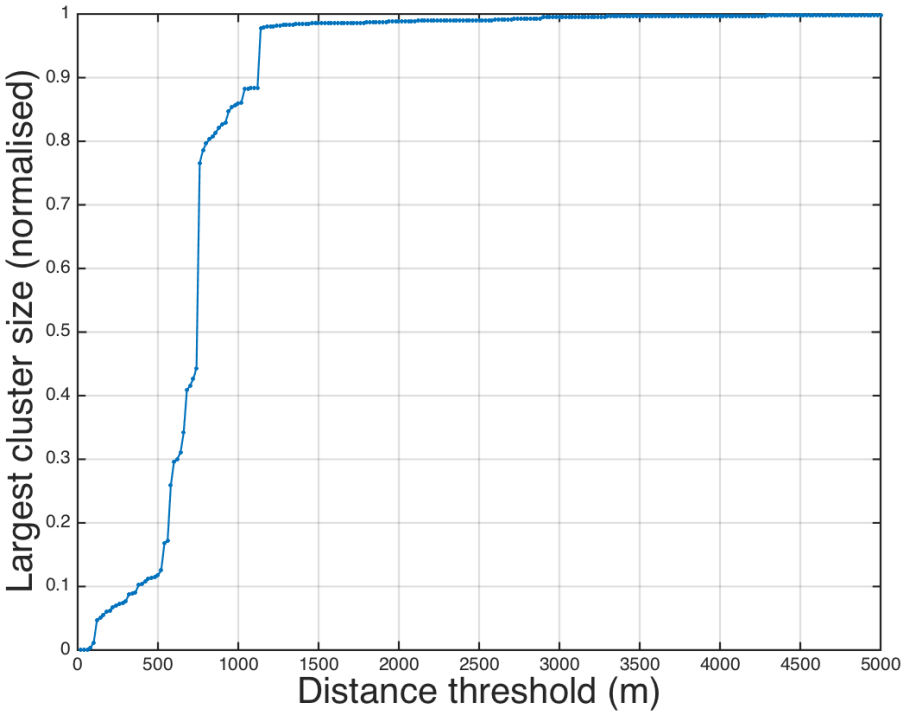
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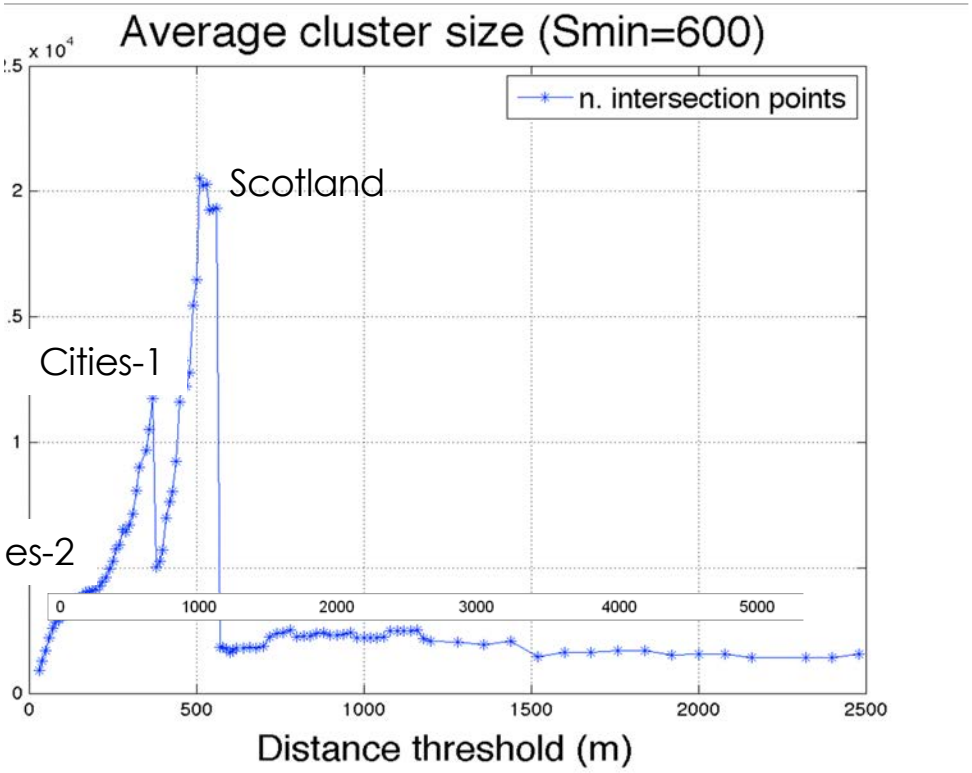


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A Hierarchy of Cities: Qualitative Transformations with Scale


Largest cluster size (normalised)
Distance threshold (m)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000


Average cluster size (Smin=600)
x 10^4
n. intersection points
Scotland
Cities-1
es-2
0
1000
2000
3000
4000
5000
0
500
1000
1500
2000
2500
Distance threshold (m)

Income and city size in Britain

- Now Geoff West, Luis Bettencourt, Jose Lobo here began to test this theory about 15 years ago and they got quite interesting results from taking the MSA data produced by the Bureau of Commerce here for 357 metro areas.
- They demonstrated super-linear scaling with an alpha parameter of about 1.0

$$Y_{city} \sim P_{city}^{\alpha} = kP_{city}^{1.1}$$

- Many issues – definition of city areas, the nature of the scaling not per head but total income and population, the aggregate nature of population

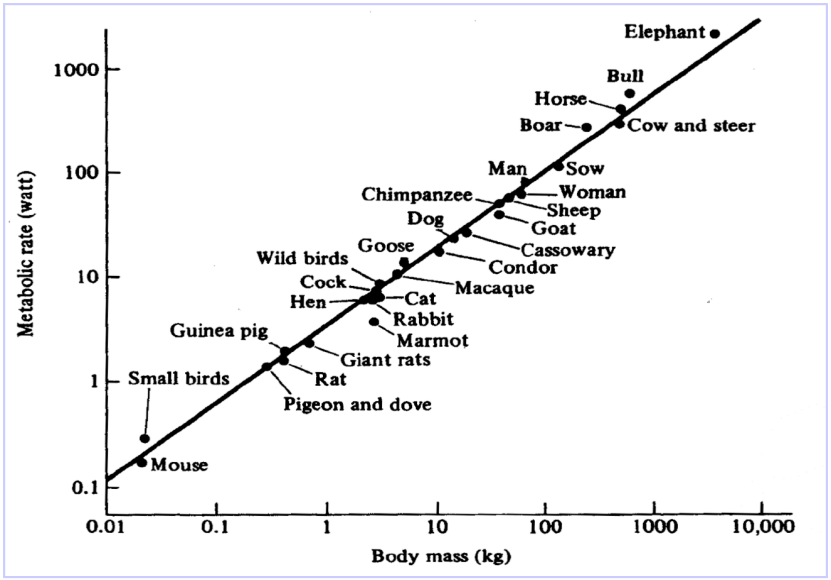
PNAS

Growth, innovation, scaling, and the pace of life in cities

Luís M. A. Bettencourt*†, José Lobo‡, Dirk Helbing§, Christian Kühnert§, and Geoffrey B. West*¶

*Theoretical Division, MS B284, Los Alamos National Laboratory, Los Alamos, NM 87545; ‡Global Institute of Sustainability, Arizona State University, P.O. Box 873211, Tempe, AZ 85287-3211; §Institute for Transport and Economics, Dresden University of Technology, Andreas-Schubert-Strasse 23, D-01062 Dresden, Germany; and ¶Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, NM 87501

Edited by Elinor Ostrom, Indiana University, Bloomington, IN, and approved March 6, 2007 (received for review November 19, 2006)



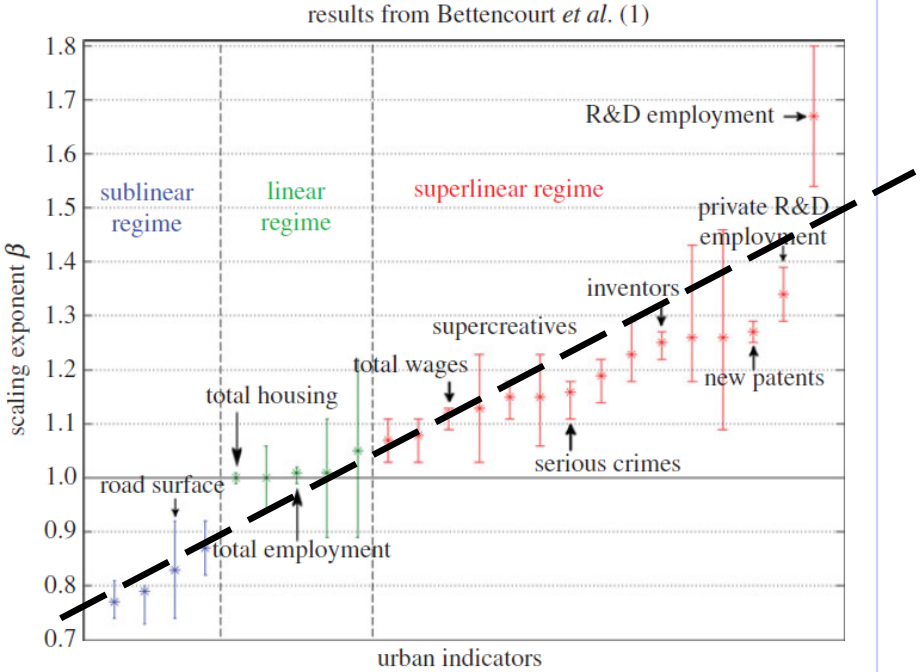


Figure 1. Exponents with 95% CI for different urban indicators found for the USA, Germany and China in reference [14]. These are colour-coded according

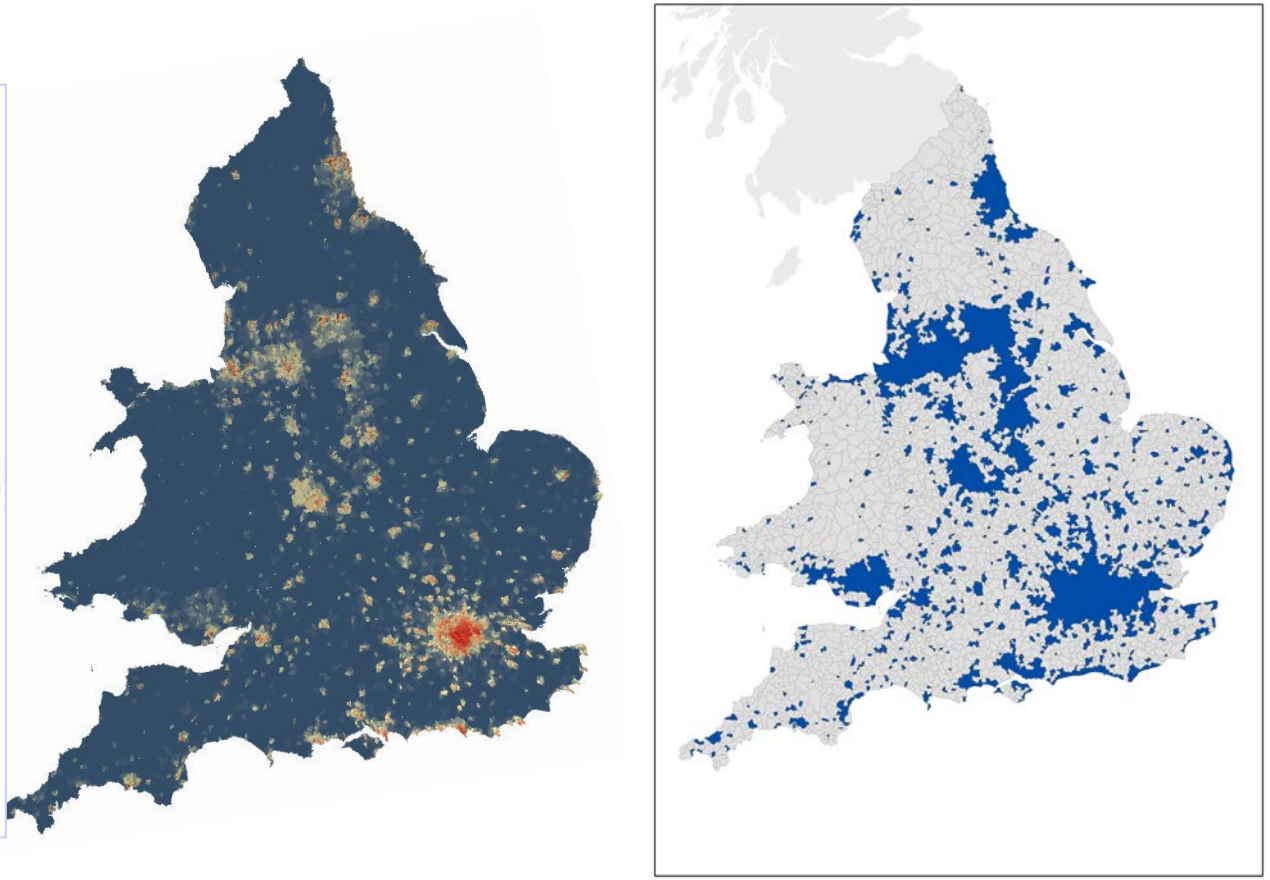
- Our group considers the West-Bettencourt results to be problematic for the UK. We do not think that income scales superlinearly with population. We cannot reproduce this result from standard definitions of UK cities and we believe that the *globalisation of cities* destroys this kind of simplicity.

- In short, everything turns on what we define as a city. In terms of MSAs, many are simply not good definitions of where people ***locate*** & particularly ***interact*** to do city things. So what we are doing is ***defining cities*** and to do this, we consider percolation theory a good potential approach. Let us begin however with some standard tests of this hypothesis for the UK or rather E & W

There are some poor definitions of cities in the EU such as LUZ – large urban zones and we should not use these but we will use density then commuting thresholds

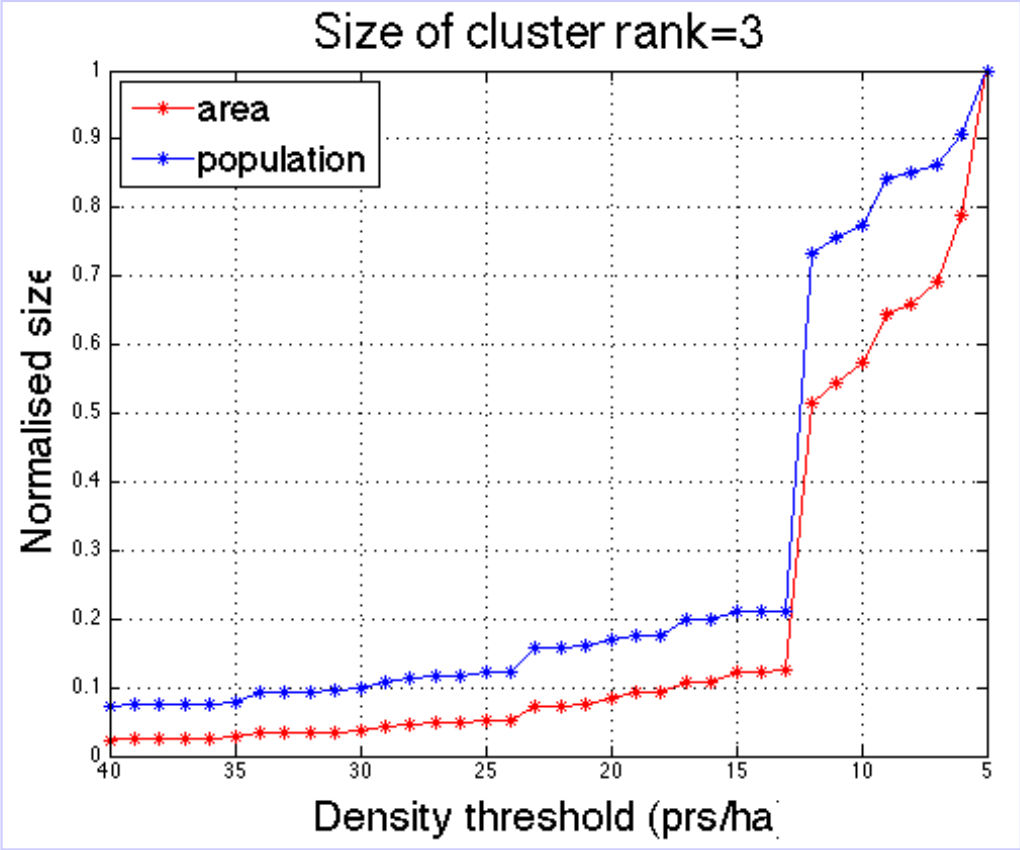


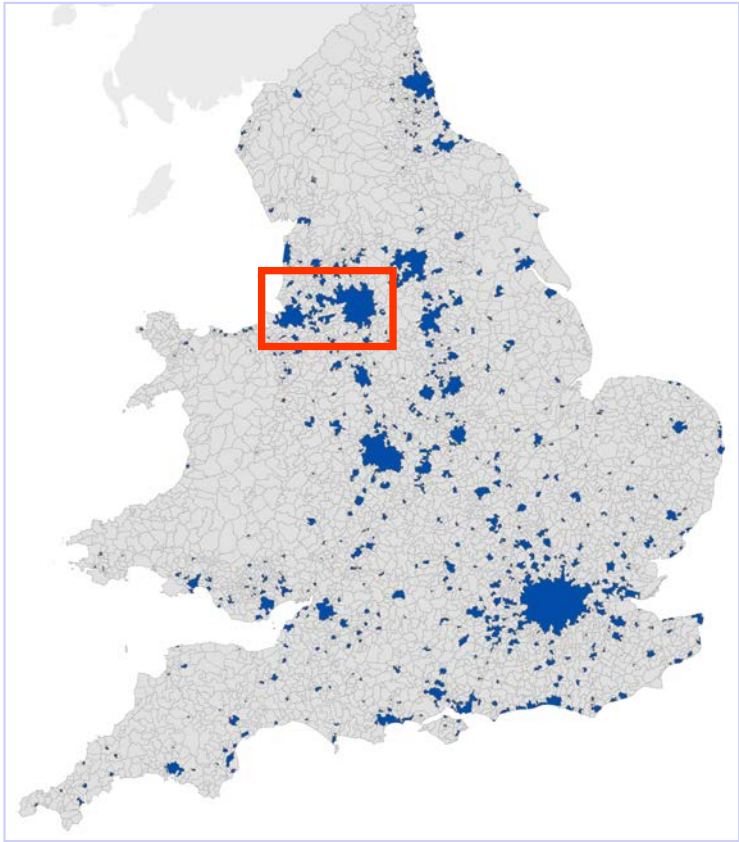
LUZs



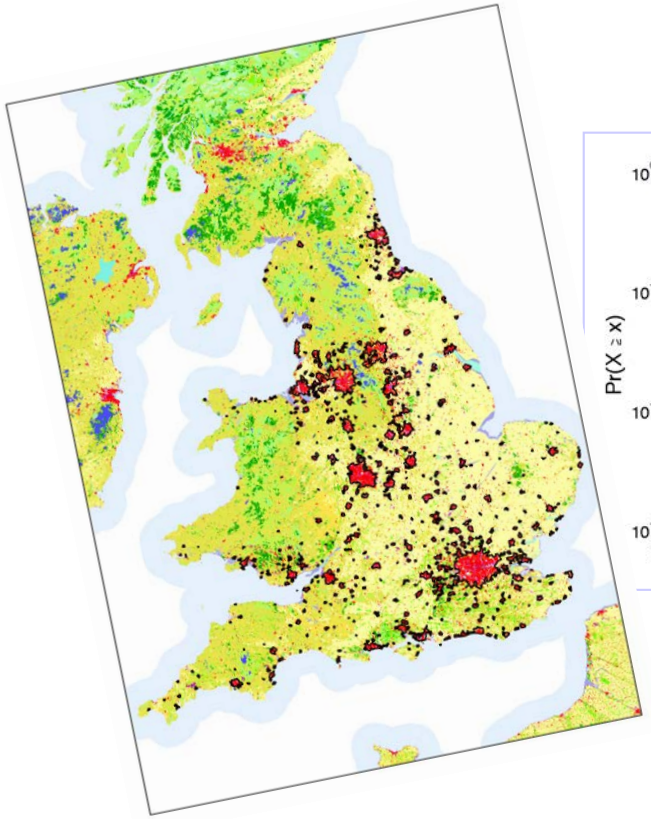
Population Density and Different Thresholds

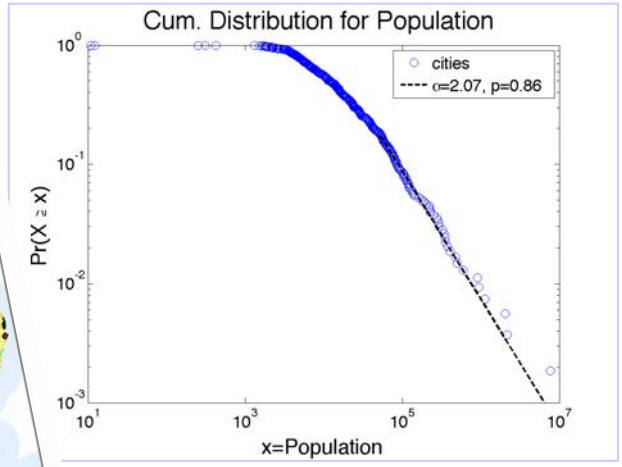
We consider 14 persons per hectare to be the right density cut-off; here Liverpool and Manchester separate

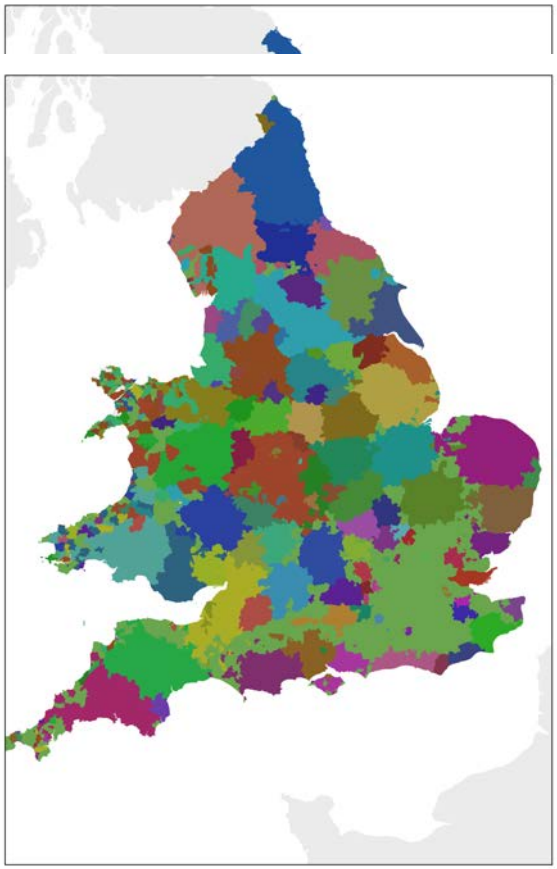




The correlation with the EU Corine RS data is good and Zipf's Law looks stable but we need to look at commuting fields – as they get larger around any place then the cities get bigger and less well-defined







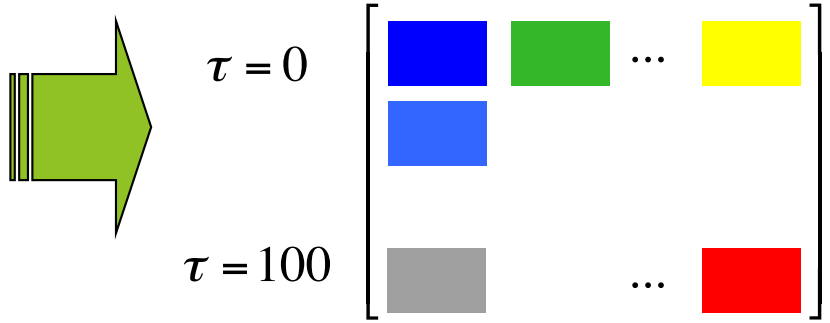
- When we put all these definitions together defining cities at ***14pph and with the 30% commuting*** thresholds, and we examine a range of indicators wrt to city size, ***we do not get superlinear scaling*** for income against the population cities.

- The bigger the city does not mean the richer in E&W and London is an enormous outlier.

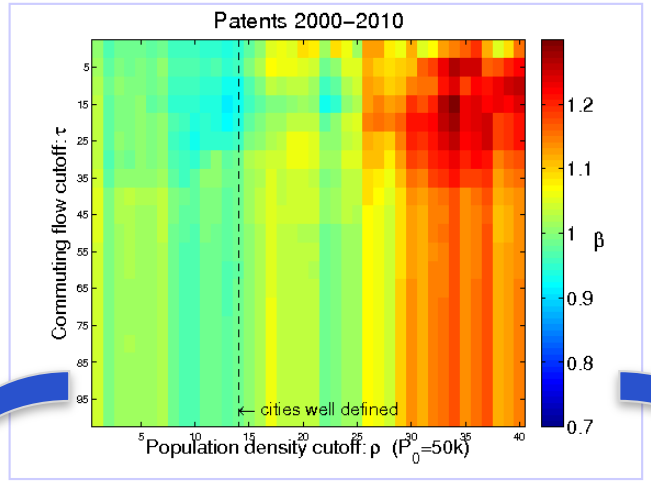
- We show these scaling relations and they are quite different from the Bettencourt-West results

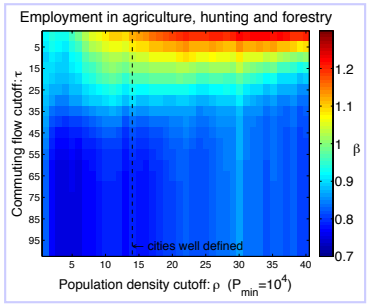
We now need to do sensitivity analysis on these definitions varying the density ρ and commuting cut-offs τ and seeing what happens to β

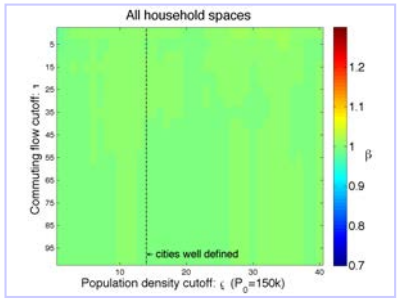
$$\begin{array}{c} \\ \tau = 0 \\ \\ \tau = 100 \end{array} \begin{array}{c} \begin{array}{cccc} \rho = 1 & & \ldots & \rho = 40 \end{array} \\ \begin{bmatrix} \beta_{11} & \beta_{12} & \ldots & \beta_{1n} \\ \beta_{21} & & & \\ & & & \\ \beta_{n1} & & \ldots & \beta_{nn} \end{bmatrix} \end{array}$$

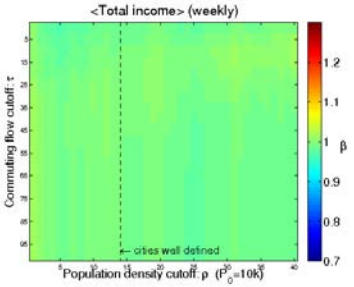


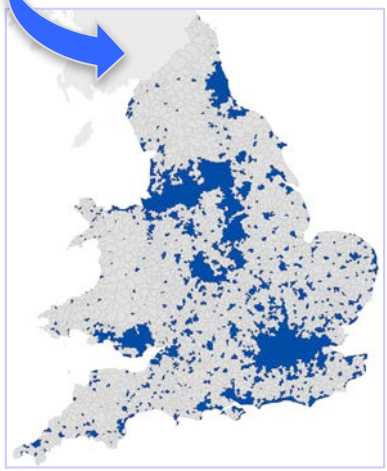
Warm colors – high and superlinear are reds to colder colors – low and sublinear blue



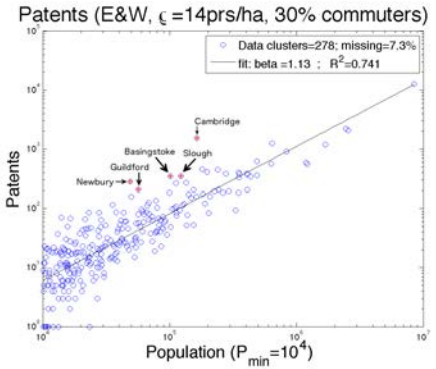


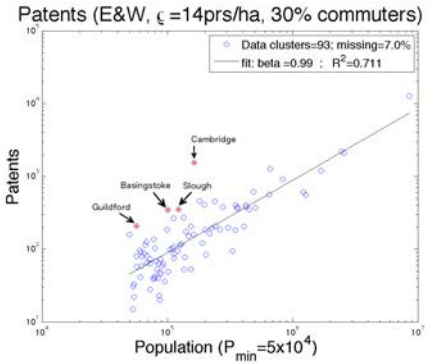






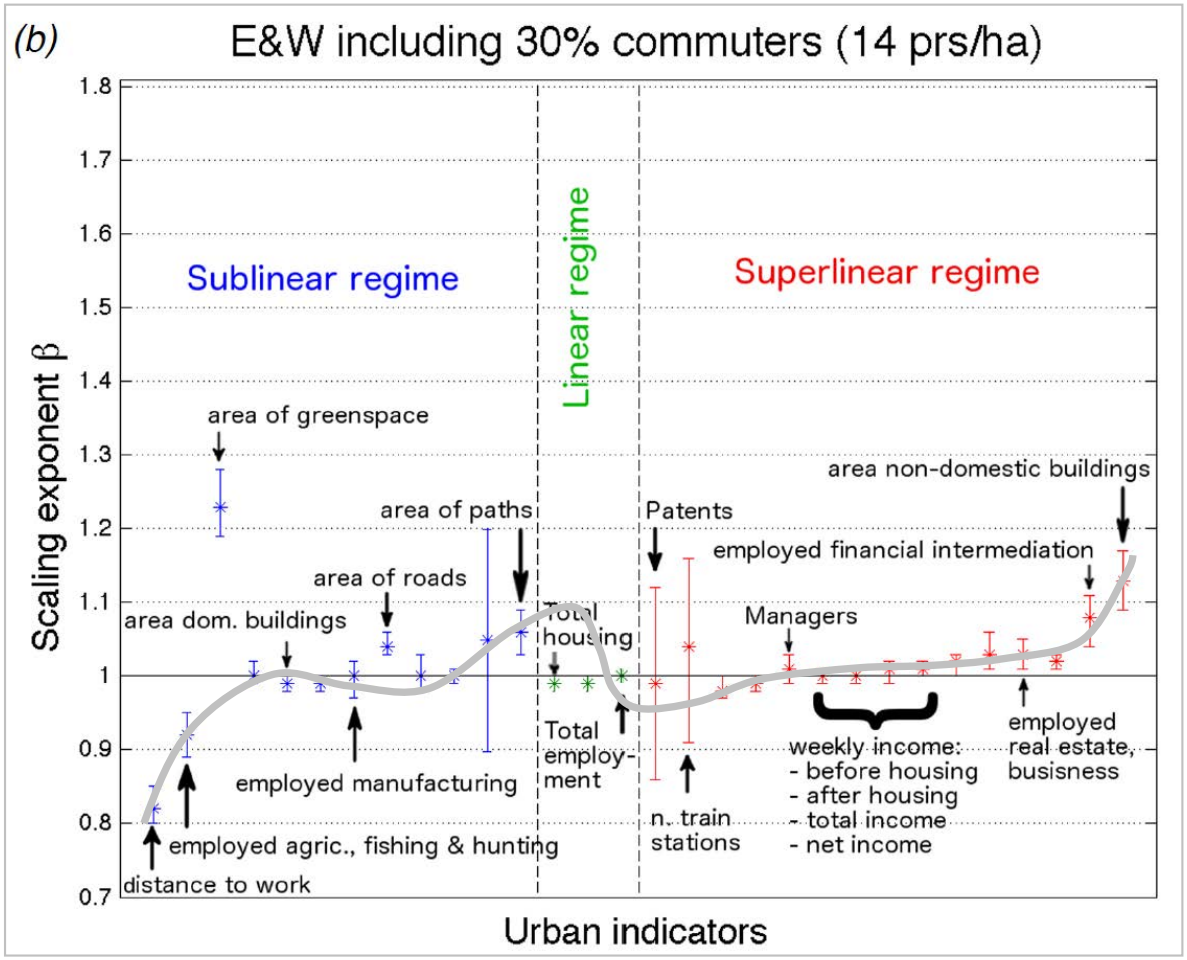




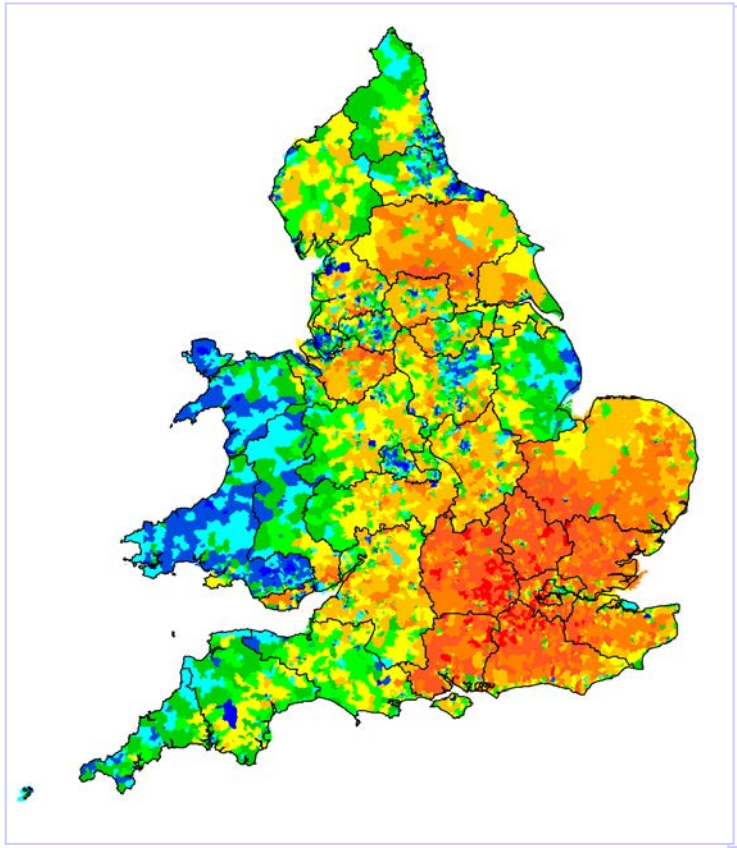


A Hierarchy of Cities: Qualitative Transformations with Scale

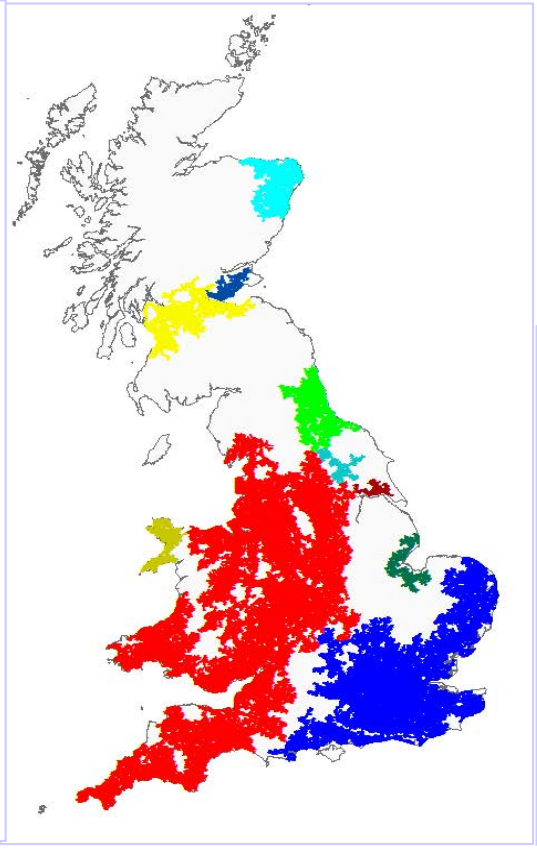
Here is the regime equivalent to the West-Bettencourt analysis and as you the linearity of the indicators is not what the W-B results for the USA imply



Do Bigger Cities Generate More Wealth?

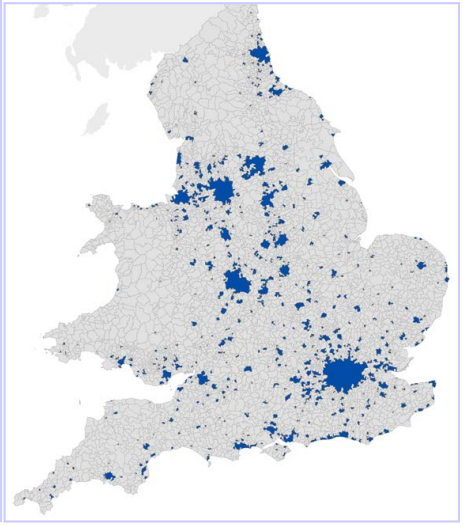


Income



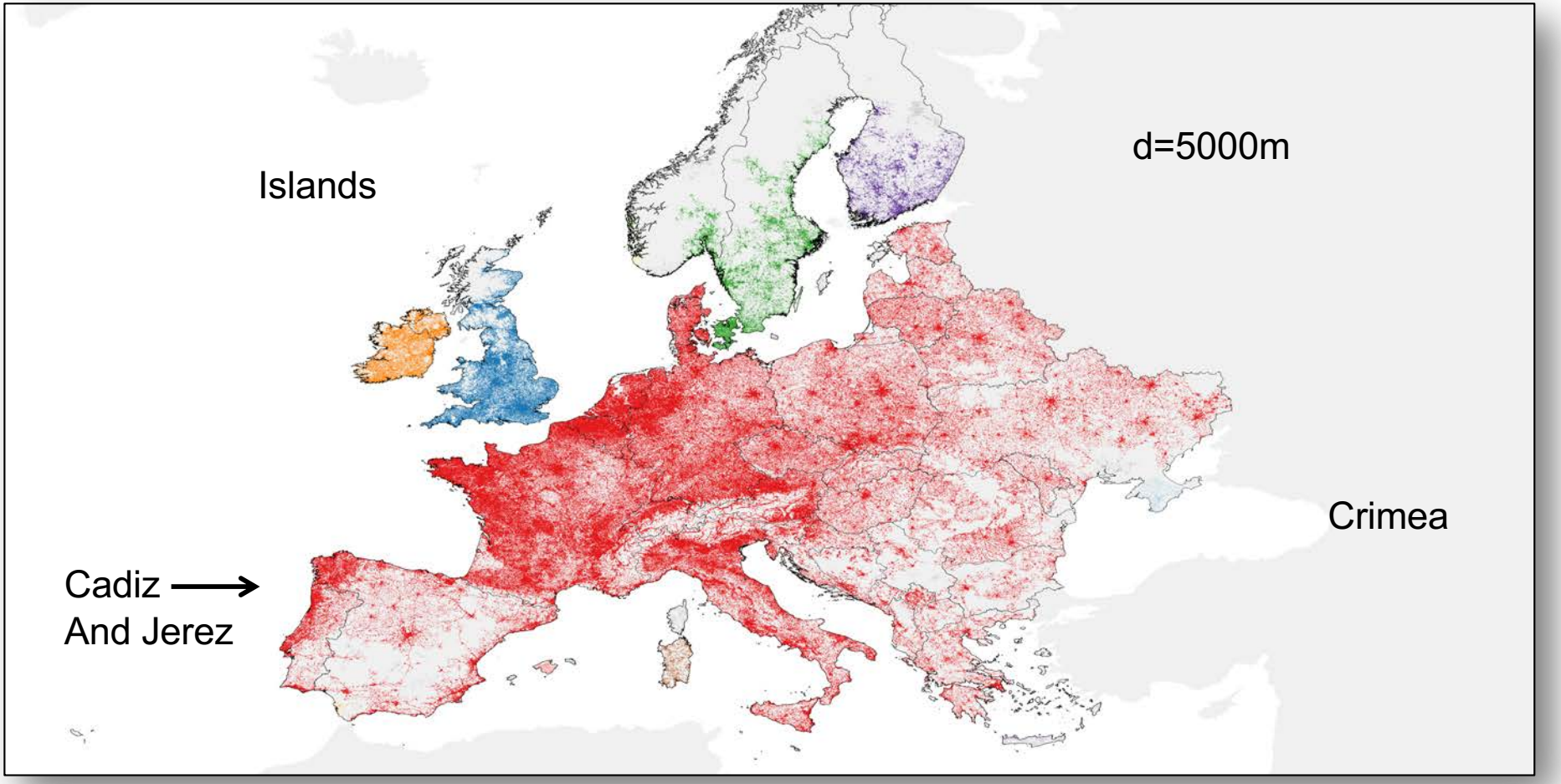
Percolation

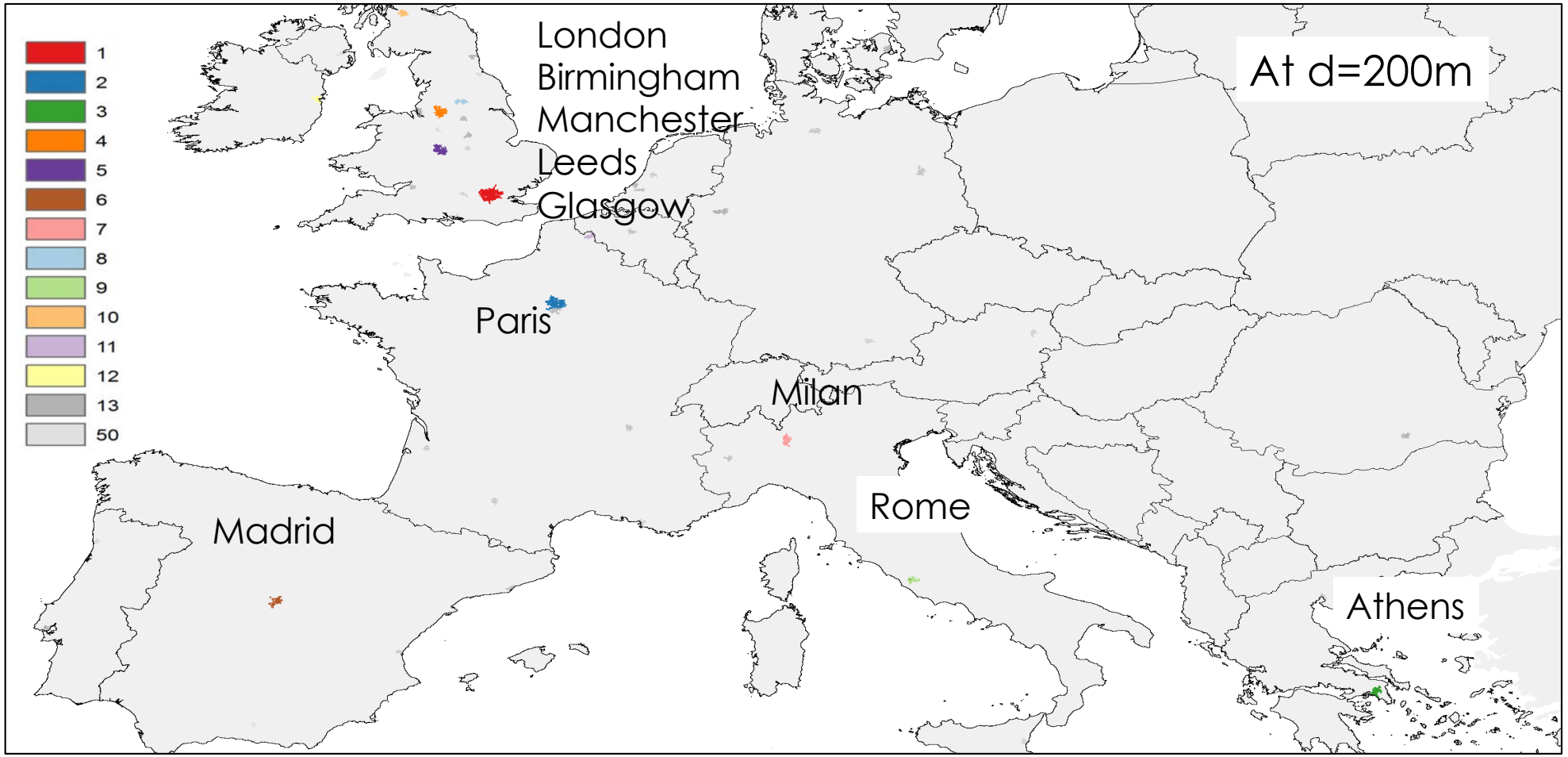
From 14 pph and 20% commuting field



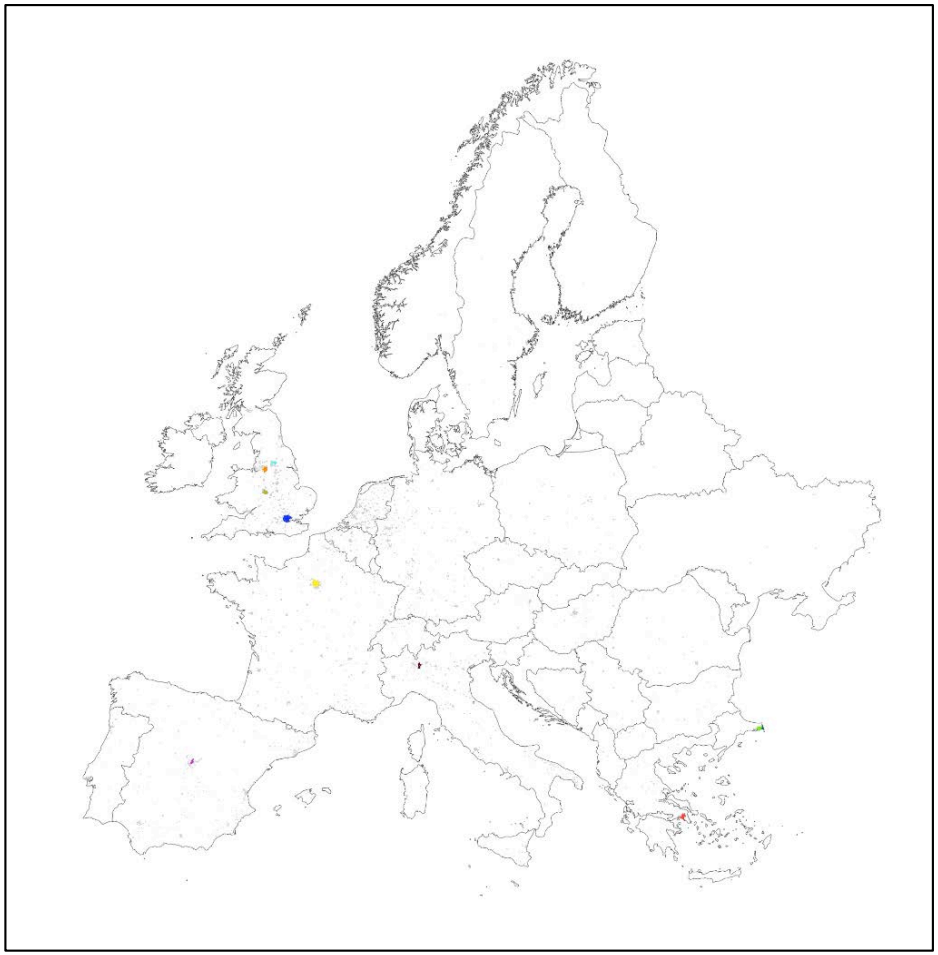
Best Cities

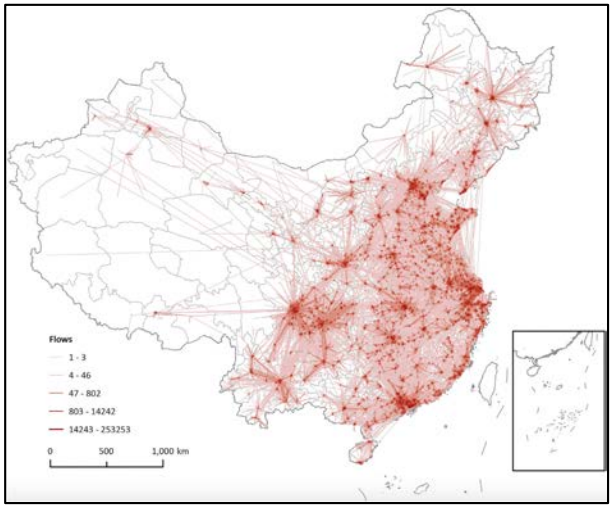
Extensions: Europe, integration and fragmentation

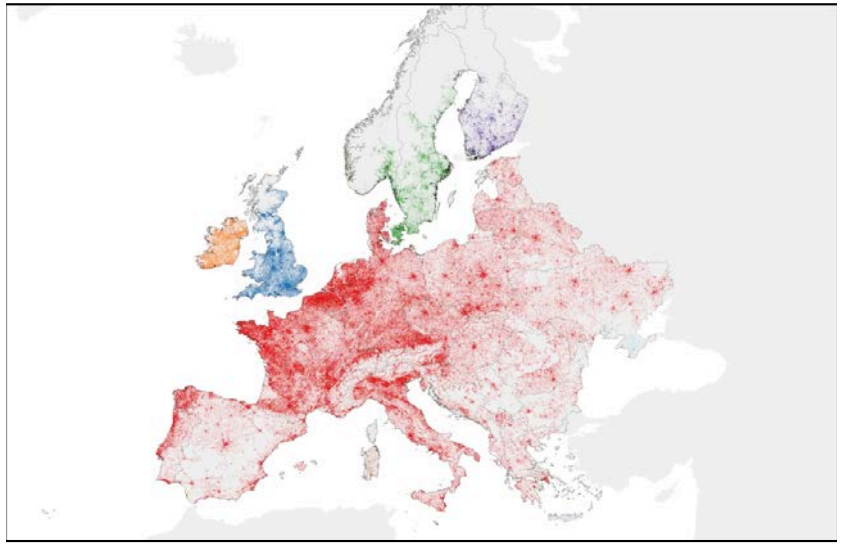




A Hierarchy of Cities: Qualitative Transformations with Scale





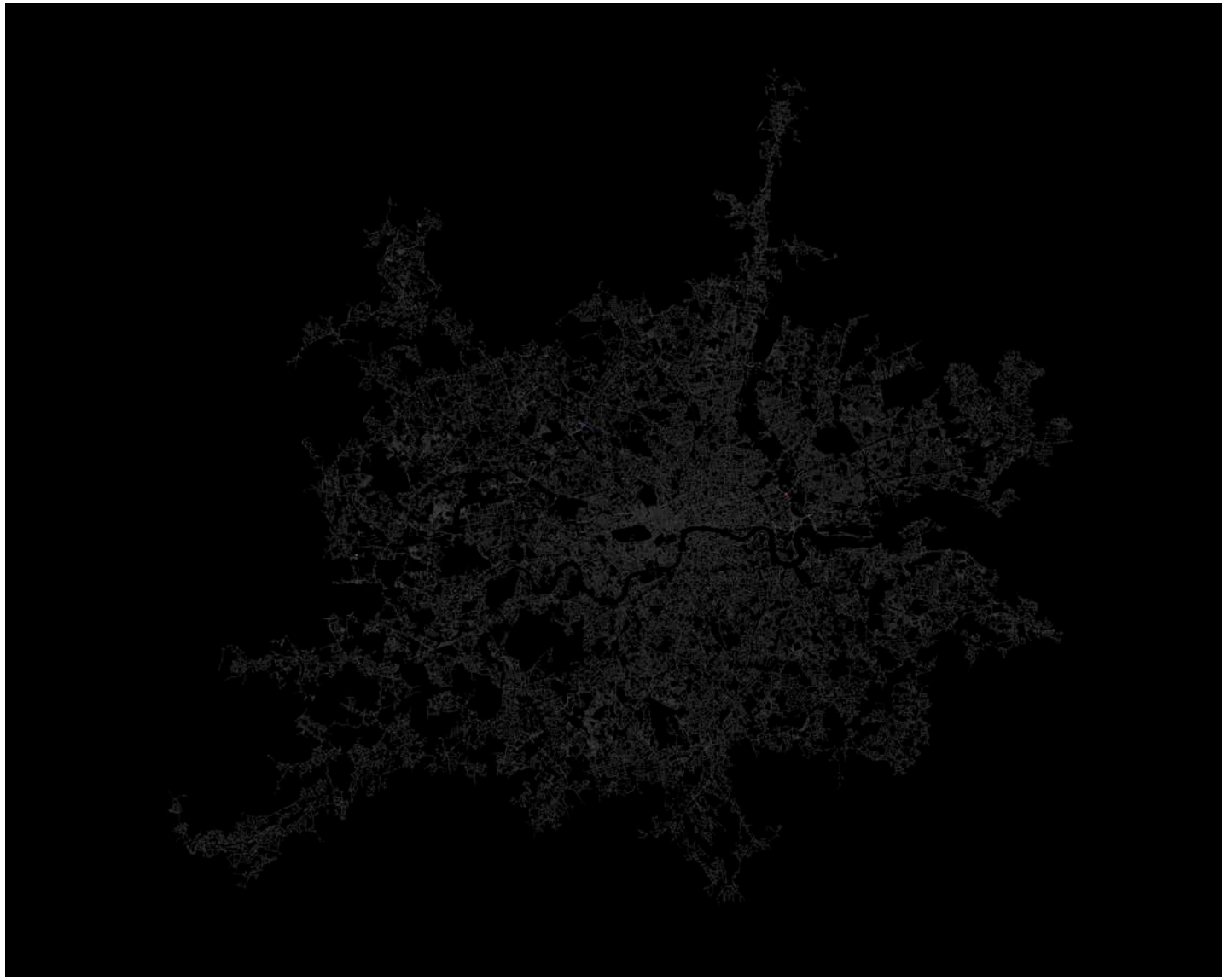


A Hierarchy of Cities: Qualitative Transformations with Scale

Smaller scales: inside the city

Let us generate clusters this time from the bottom up, from a very local threshold up to the radius of the entire city. We are looking here at London within the M25 – the GLA area for purely data availability reasons – we should go wider – include Oxford?

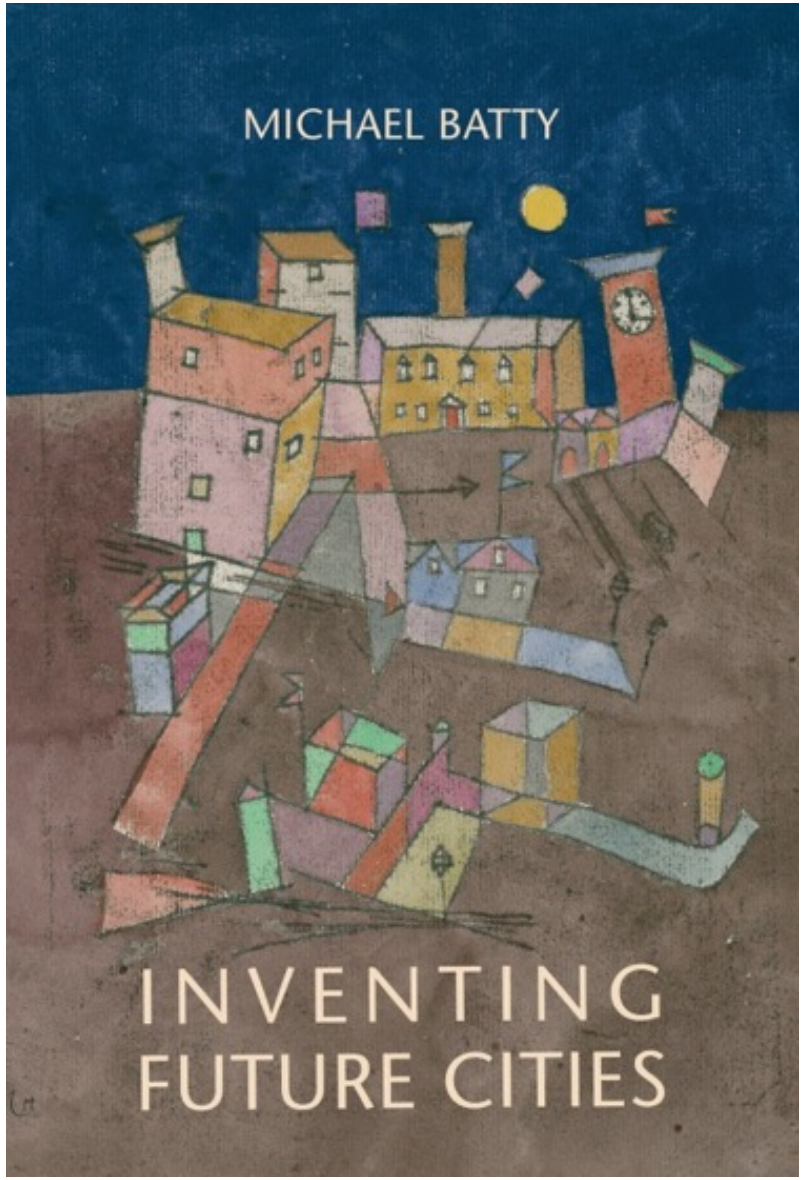




A Hierarchy of Cities: Qualitative Transformations with Scale

Some Brief Conclusions

- To an extent, all this is hardly new but it focuses on something we have been very bad at doing - looking at the properties of cities across scales
- We now have a lot of work going on about these properties – partly it is addressed by looking at elements and how they vary with size and shape – how they scale – scaling – rank size, power laws
- But it is also about how cities vary in their patterns of interaction. How we travel differently as cities get bigger etc.
- What we need is a better worked out theory relating the methodological properties of scale to their theoretical meanings



MIT Press, 2018.

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Cities and regions in Britain through hierarchical percolation

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Urban systems present hierarchical structures at many different scales. These are observed as administrative regional delimitations which are the outcome of complex geographical, political and historical processes which leave almost indelible footprints on infrastructure such as the street network. In this work, we uncover a set of hierarchies in Britain at different scales using percolation theory on the street network and on its intersections which are the primary points of interaction and urban agglomeration. At the larger scales, the observed hierarchical structures can be interpreted as regional fractures of Britain, observed in various forms, from natural boundaries, such as National Parks, to regional divisions based on social class and wealth such as the well-known North–South divide. At smaller scales, cities are generated through recursive percolations on each of the emerging regional clusters. We examine the evolution of the morphology of the system as a whole, by measuring the fractal dimension of the clusters at each distance threshold in the percolation. We observe that this reaches a maximum plateau at a specific distance. The clusters defined at this distance threshold are in excellent correspondence with the boundaries of cities recovered from satellite images, and from previous methods using population density.

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